Design of trajectory and perturbation analysis for satellite orbital parameters

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ABSTRACT.

The aviation and space industry is advancing at a fast pace. In light of the launch and re-entry accidents that have happened in the past it is essential to have accurate analysis of the parameters required for error free launch and placement of the launched body in correct trajectory. In this paper we discuss various orbital parameters required for placing a satellite, launched from earth, in correct orbit and the designing of its trajectory using Patched Conic Approximation method. Also presented are various perturbations factors like lunar gravity and atmospheric drag. The effect of these factors is considered and compared with the ideal cases.

Keywords: Keplerian elements, perturbation, patched conic approximation method, atmospheric drag.

1 INTRODUCTION

The SSETI (Student Space Exploration and Technology Initiative) paper started to create and build a micro-satellite. Also it should be completed with the development of a Moon Rover in the third mission. The launch described in this paper is the micro satellite. The goal of this launch is to make the ESMO satellite orbit in to Moon. One of the teams work on control of the attitude and the orbit of the ESMO satellite. To add control to the orbit of the satellite, forces acting on the satellite need to be described. The number of celestial bodies has to be decided by making the problem a two-, three- or four-body problem depending on the number of celestial bodies included. These will form the largest forces and other forces such as atmospheric drag and solar radiation pressure can also be included. Also, there are many possibilities from amongst possible trajectories to get to the Moon. Some are more fuel-efficient than others, but these often use longer time. But no matter which is used, there will always be perturbations and combinations to consider. Safety is the probability of causing injury or loss of life. Unreliable launchers are not necessarily unsafe, whereas reliable launchers are usually, but not invariably safe. Apart from catastrophic failure of the launch vehicle itself other safety hazards and Van Allen radiation belts which preclude orbits which spend long periods within them. Trajectory optimization is the process of designing a trajectory that minimizes or maximizes some measure of performance within prescribed constraint boundaries. While not exactly the same, the goal of solving a trajectory optimization problem is essentially the same as solving an optimal control problem. This problem was first studied by Robert H. Goddard and is also known as the Goddard problem.

a. The two body problem

The simplest of the n-body problems is the two-body problem, only two masses is considered at a time. Let the masses be denoted by m_1 and m_2.

$$\ddot{r_2} - \ddot{r_1} = -G(m_1 + m_2)\frac{r_1 - r_2}{r3}$$

This is the equation of motion for the 2-body problem

2 DYNAMICS OF ORBITS

When orbital mechanics is to be described, there are many different types of coordinate systems to choose from. It is quite easily expressed in polar coordinates. The plane polar coordinates are (r, θ) and the unit vectors are
as shown in Figure.1. [1] The velocity vector is

$$\vec{v} = \dot{r}\overrightarrow{e_r} + r\theta\overrightarrow{e_\theta}$$

And acceleration vector is

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\overrightarrow{e_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\overrightarrow{e_\theta}$$

The equations of motion can be divided up into radial and transverse direction

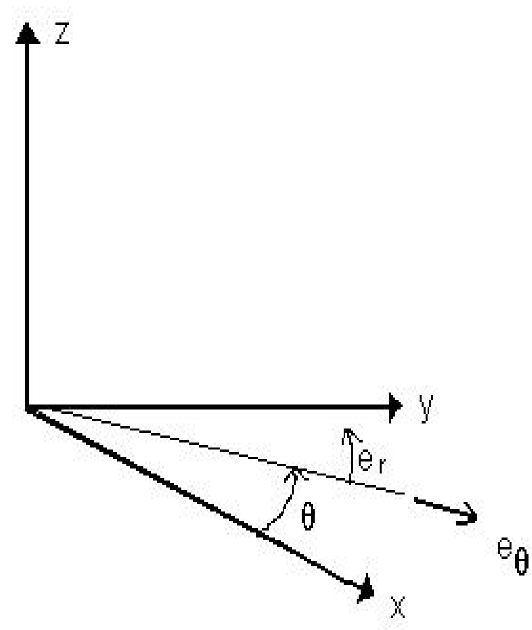

Fig 1: Polar Coordinates

In the radial direction the equation of motion is

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2}$$

Where μ = Gm (G - gravitational constant, m - mass of spacecraft) and the whole expression on the right hand side is gravity.This is the only acceleration that works in radial direction.

$$r(\theta) = \frac{\frac{h^2}{\mu}}{1 + \frac{Ah^2}{\mu}\cos(\theta - \theta_0)}$$

Where A and θ_0 are constants. This is polar coordinates of an ellipse equation.

2.1 Orbits geometry

The simplest orbits follow basic geometry of conic sections. Conic sections are different intersections of a plane and a cone. The circle intersects the cone horizontally, and the ellipse intersects the cone with a tilt, see Figure 2. Both are closed curves. The hyperbola intersects the cone resulting in an open curve. There is yet another basic conic section; the parabola. [2] The parabola is the single curve which divides the closed ellipse from the open hyperbola. Here the plane is parallel to the side of the cone. There are two points of particular interest on the orbits; the epicentre and the epicentre. The epicentre is the point where a spacecraft will be closest to the object it is orbiting, and the epicentre is the point furthest away b semi-minor axis, a semi-major axis, ae distance from the centre to the focal point, the distance is determined by the conic section of eccentricity e.

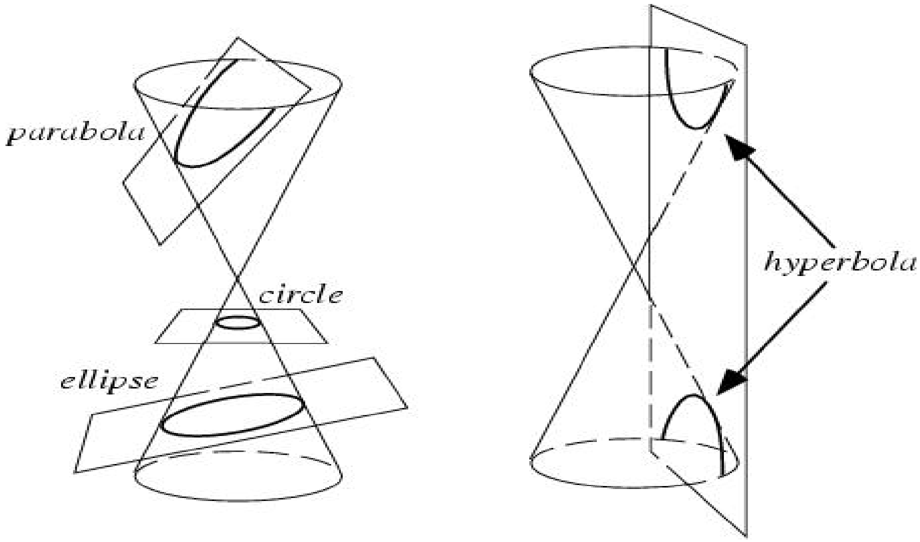

Fig 2: Conic sections

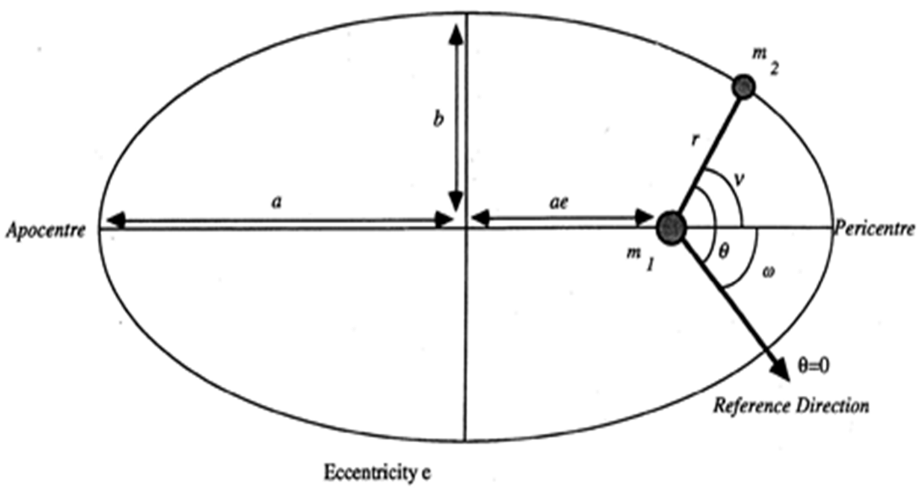

Fig 3: Orbital parameter

2.2 Elliptical orbits

The orbit period can be calculated from the equation for the area of an ellipse, the definition of an orbit period and and results in

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

2.3 Cicrcular, parabolic and hyperbolic orbits

In circular orbit, the eccentricity is zero, which means that the radius is constant; R. This results in the following velocity and orbit period equations

$$v = \sqrt{\frac{\mu}{R}}$$

$$T = 2\pi\sqrt{\frac{R^3}{\mu}}$$

In a parabolic orbit the eccentricity is one. This results in the velocity equation

$$v = \sqrt{\frac{2\mu}{r}}$$

The orbit period T → ∞ since a → ∞.
In hyperbolic orbit the eccentricity is greater than one. [3] The velocity equation is then

$$v^2 = 2\frac{\mu}{r} + V^2{}_\infty$$

Where V1 is the hyperbolic excess speed expressed as

$$V_\infty = \sqrt{\frac{\mu}{a}}$$

3 PERTURBATIONS OF ORBITS

3.1 The flattening of the earth

Earth is in everyday life thought of as being a perfect sphere. But this is not entirely true. Earth is slightly flattened at top and bottom.

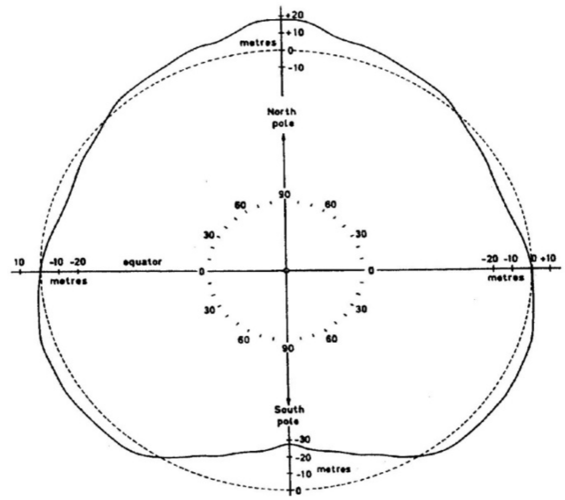

Fig 4: Flattening of the earth

Besides being flat at top and bottom, Earth has a bulge on Equator. It is not important to take this effect into account for Low Earth Orbits (LEOs) as it will avarage out after many revolutions, but it should be taken into account when determining orbits for Geosynchronous Earth Orbits (GEOs). As the ESMO satellite will keep a high altitude orbit around Earth before being launched into Moon orbit, it is relevant. [4]

3.2 Atmospheric drag

As we are considering LEO satellite atmospheric drag dissipates energy from the satellite in orbit. The orbital height of the satellite will reduce slightly. It is inversely proportional to air density. Air density decreases with rise in altitude. [5] The ESMO satellite will be in an orbit where atmospheric drag is relevant and possibly in the start of the transfer orbit.

The drag force FD on a body acts in the opposite direction of the velocity vector and is given by the equation

$$AD_{drag} = \frac{1}{2}\rho v^2 A c_{d_{co}}$$

Where, AD_{drag} is the drag force acting on the satellite,
ρ is the density of atmosphere at that level
v is velocity of satellite
A is frontal Area of Satellite,
Cd_{co} is the Drag coefficient

3.3 Solar and lunar gravity perturbation

This solar and lunar perturbation causes tidal forces that perturb the satellite from its orbit. [6] The formulae for the perturbation calculation due to solar and lunar gravity are given as follows:

$$\Omega moon = -\frac{0.00338\ \cos(i)}{n}$$

$$\Omega sun = -\frac{0.00154\cos(i)}{n}$$

$$\omega moon = \frac{0.00169\,(4 - 5sin^2(i))}{n}$$

$$\omega sun = \frac{0.00074\,(4 - 5sin^2(i))}{n}$$

Where,
i -- Orbit inclination,
n -- Number of orbit revolutions per day,
Ω And ω -- degrees per day.

4. TRAJECTORIES

4.1 Hohmann transfer

The Hohmann Transfer is the traditional way for constructing a satellite transfer to the Moon. It uses two-body dynamics, and is constructed by determining an elliptic transfer of orbit from an Earth parking orbit to the Moon's obit. [7] It is an expensive approach, when the ratio of the two radii of the orbits is large as it requires a large velocity. This subsection will therefore only describe it briefly.

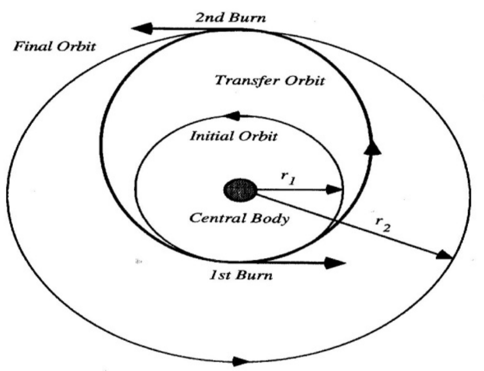

Fig 5: Hohmann Transfer

4.2 Patched conic approximation method (PCA)

The Patched Conic Approximation is a well-known method. When used on a transfer between Earth and the Moon, it is also referred to as the Lunar Patched Conic. [8] It is a good way to make an approximation of a simulation of a lunar transfer orbit. Still it is restricted to the two-body problem, but more than one two-body problem are used, hence the name of the method.

Well beyond the orbit of the Moon, so the patched conic method is a rough approximation.

1. Earth departure; Earth's gravitational pull dominates
2. Arrival at the Moon; Moon's gravitational pull dominates

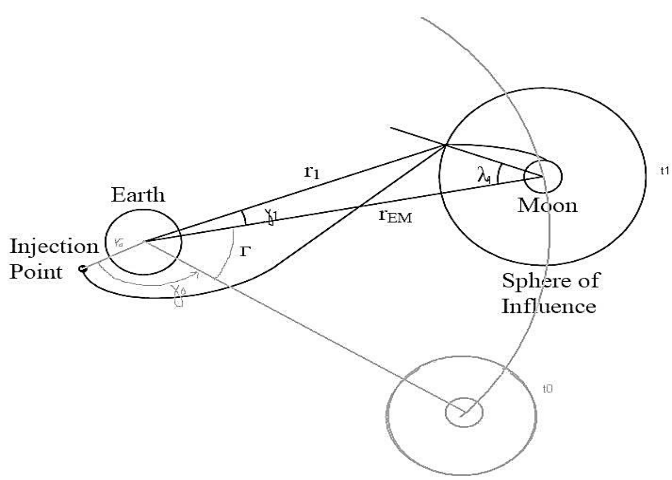

Fig 6: Patched Conic Approximation Method

This trajectory is a Hohmann-transfer ellipse around the Sun. The Hohmann transfer was described in Section 3.2.1. In the second region, motions are relative to Earth. This is really the first part of the trajectory. Here, the satellite escapes Earth and arrives at the SOI with the required velocity to enter into the heliocentric transfer orbit of region one. The satellite needs to increase its velocity in the parking orbit by a certain amount. In the third region, motions are relative to the Moon. Here, the satellite needs to be slowed down. If not, it will only swing by the Moon on a hyperbolic trajectory and depart the SOI on the other side.

5 RESULTS

Satellite solar radiation impact

- Area of satellite facing the sun decreases to a certain limit (threshold limit) with respect to the decrease in deceleration due to solar radiation and after the threshold limit it increase.
- Mass of satellite facing the sun decreases to a certain limit (threshold limit) with respect to the decrease in deceleration due to solar radiation and after the threshold limit it increases.

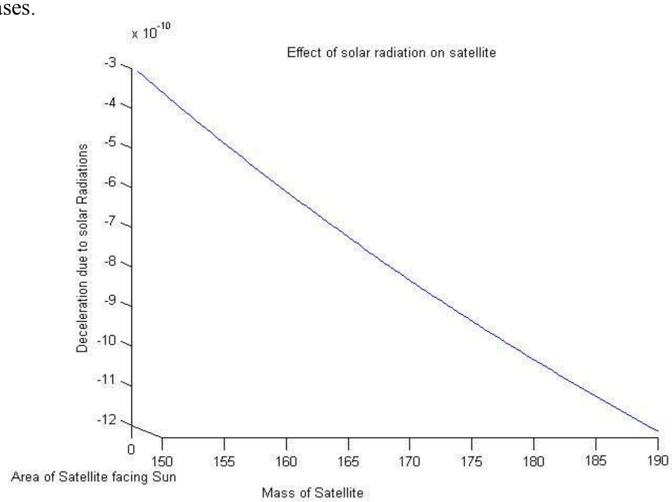

Fig 7: Solar Radiation on Satellite Mass v/s Deceleration

Effect of atmospheric drag (Fig8: changes in satellite semi major axis)

This graph is hyperbolic in nature. Ballistic constant increases with the decrease in acceleration due to atmospheric drag.
• Change in velocity per revolution due to atmospheric drag vs ballistic constant
This graph has a logarithmic decrement kind of nature. Ballistic constant increases with the decrease in change in velocity per revolution due to atmospheric drag.
• Lifetime of satellite in second's vs deceleration due to atmospheric drag
The lifetime of satellites decreases with the decrease in deceleration due to atmospheric drag
• Change in revolution period due to atmospheric vs ballistic constant
This graph is hyperbolic in nature. Ballistic constant increases with decrease in change in revolution period due to atmospheric drag.

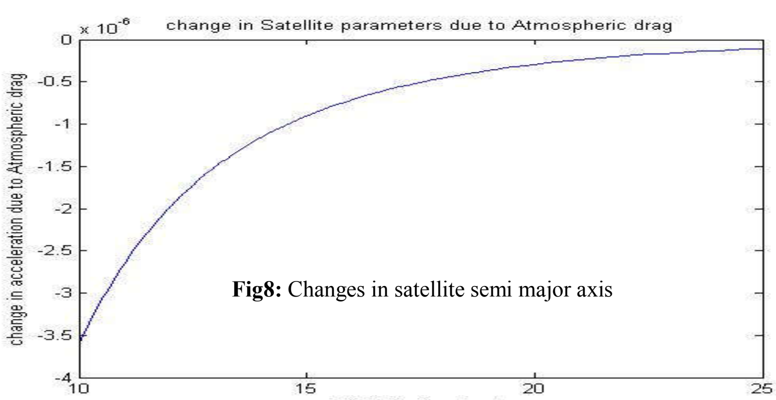

Fig8: Changes in satellite semi major axis

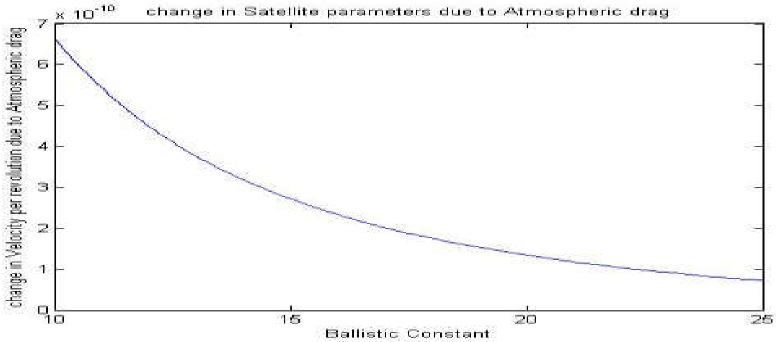

Fig 9: Change in Satellite Time Period

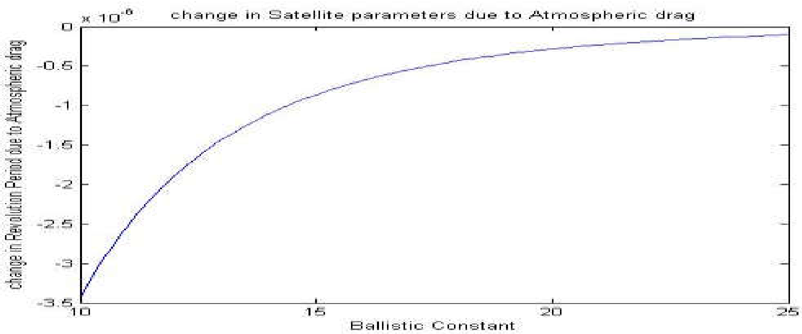

Fig 10: Change in Satellite Velocity

Perturbation due to earth shape

- Variation in longitude of ascending node with semi- major axis and angle of inclination. Semi major axis increases with increase in longitude of ascending node to a maximum point after which it tends to remain constant and same goes for angle of inclination.

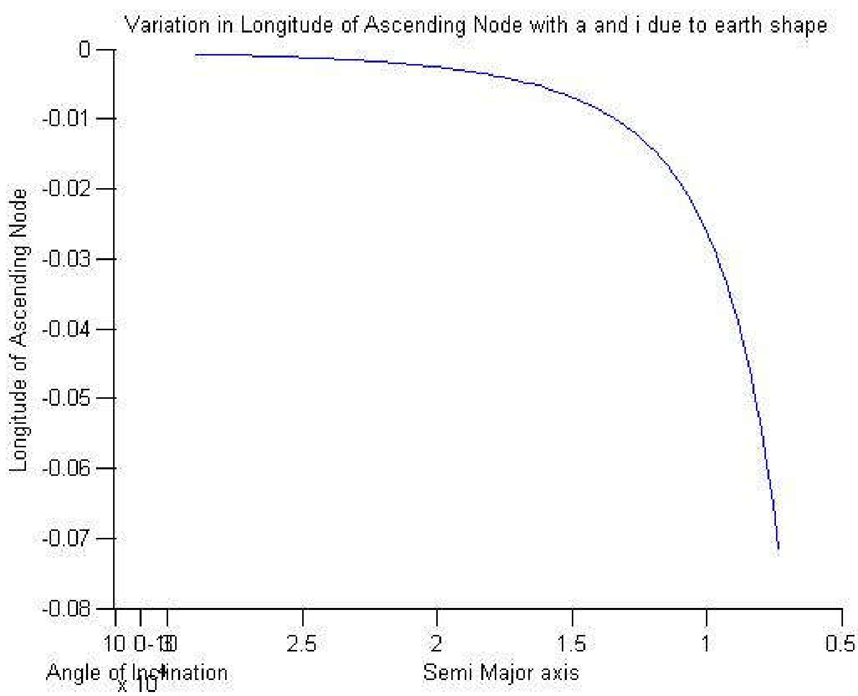

Fig 11: Variation in Ascending Node for Shape of Earth

- Variation in argument of perigee with semi- major axis and angle of inclination. Semi major axis decreases with the increase in argument of perigee to a minimum value after which it tends to remain constant and angle of angle of inclination increases with the increase in argument of perigee to a minimum value after which it tends to remain constant.

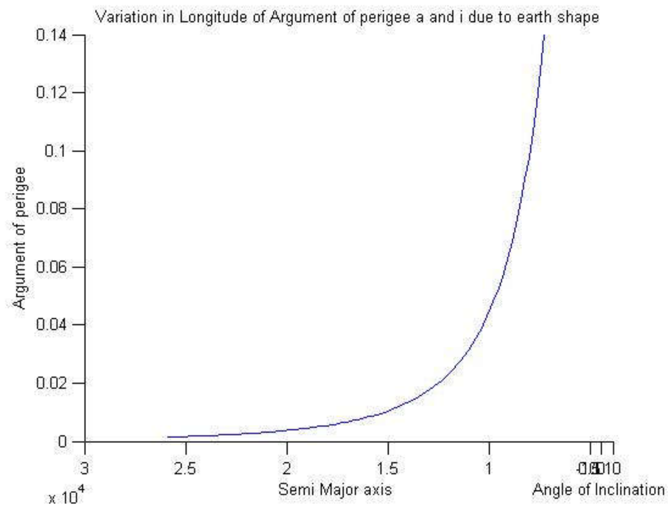

Fig 12: Variation in longitude of Ascending Node

Perturbation due to lunar and solar gravity

- Variation in longitude of ascending node with semi major axis angle of inclination due to moon Semi major axis decreases with the decrease in longitude of ascending node while angle of inclination increases.

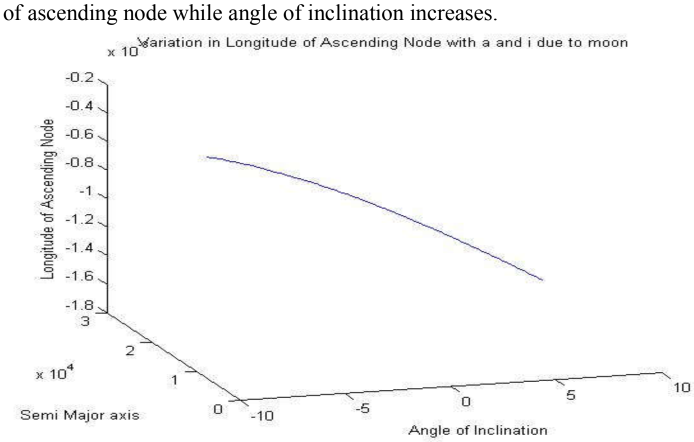

Fig 13: Variation in LAAN due to Moon

- Variation in longitude of ascending node with semi major axis angle of inclination due to sun
- Semi major axis decreases with the decrease in longitude of ascending node while angle of inclination increases.

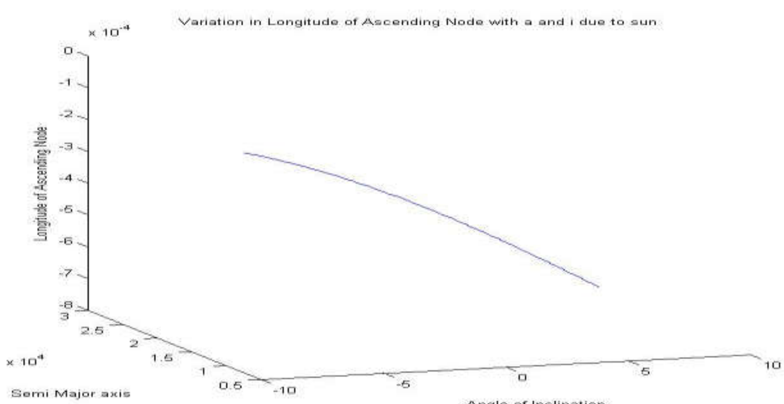

Fig 14: Variation in LAAN due to Sun

- Variation in argument of perigee with semi major axis angle of inclination due to moon Argument of perigee has no dominant effect on semi major axis while angle of inclination slightly increases with the increase in argument of perigee.

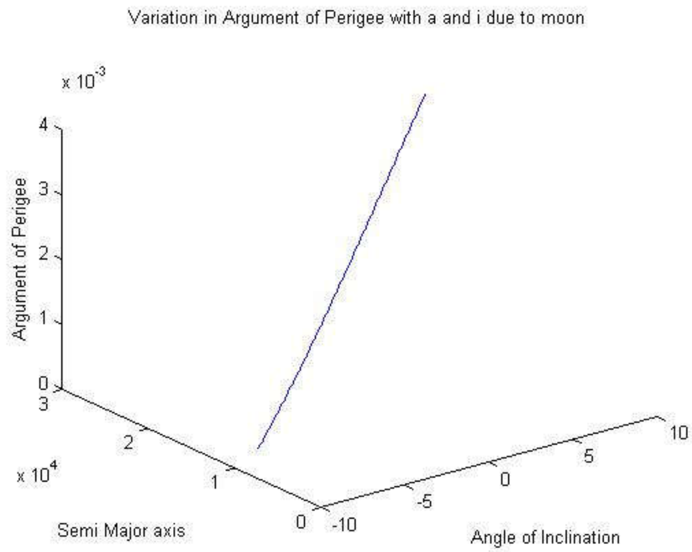

Fig 15: Variation in Argument of Perigee due to moon

- Variation in argument of perigee with semi major axis angle of inclination due to Sun
  Argument of perigee has no dominant effect on semi major axis while angle of inclination slightly increases with the increase in argument of perigee.

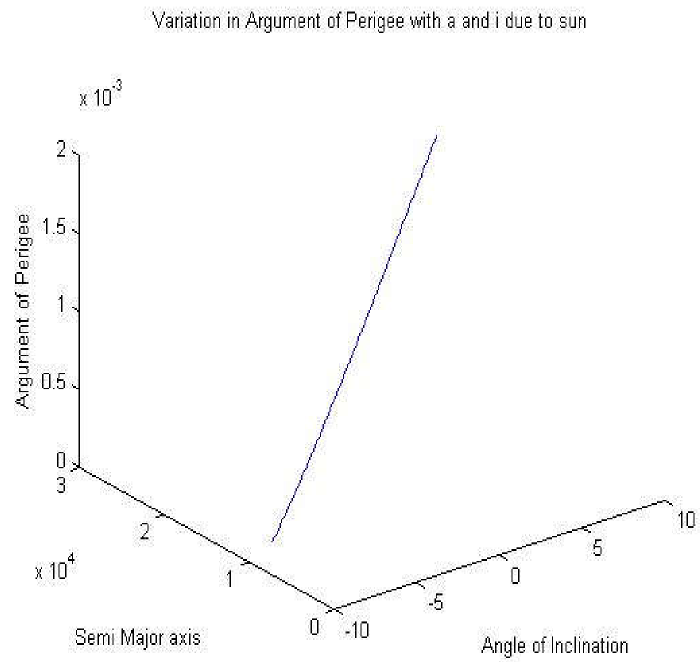

Fig 16: Variation in Argument of Perigee due to Sun

Plotting of trajectories

- **Trajectory analysis**
  As per the requirements, platform for the testing of satellite is chosen as geosynchronous orbit. As in this orbit the satellite remains constant over a particular point, so the data accumulated is more accurate and precise. Steps for reaching this orbit are:

Hohmann transfer

- First the satellite is launched in a highly elliptical orbit.
- When the satellite reaches apogee of the orbit, velocity injection is done to send the satellite in a circular orbit with firing retro rockets for velocity reduction.
- When the satellite reaches the perigee position of the circular orbit, rockets fired puts the satellite in the geo-synchronous satellite.

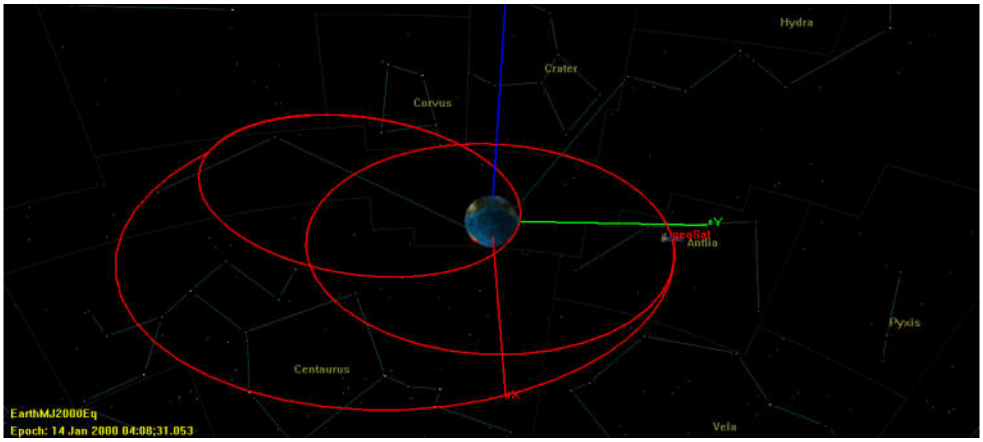

Fig 17: Hohmann Transfer

One tangent

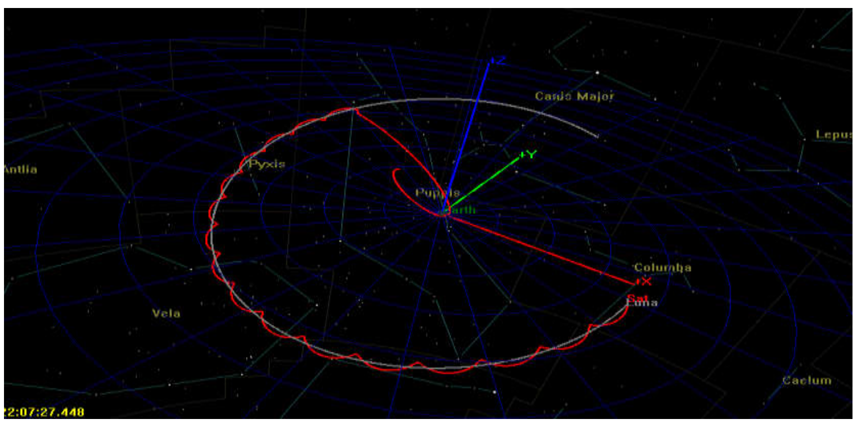

Fig 18: One Tangent Burn

Burn Moon's influence on satellite

- Rockets are fired from the perigee of the geo-synchronous orbit and hence it comes in the sphere of influence of moon.
- Due to the firing of rockets for short interval spiral transfer is actuated in the trajectory of the satellite.
- The trajectory as seen from earth is shown below:

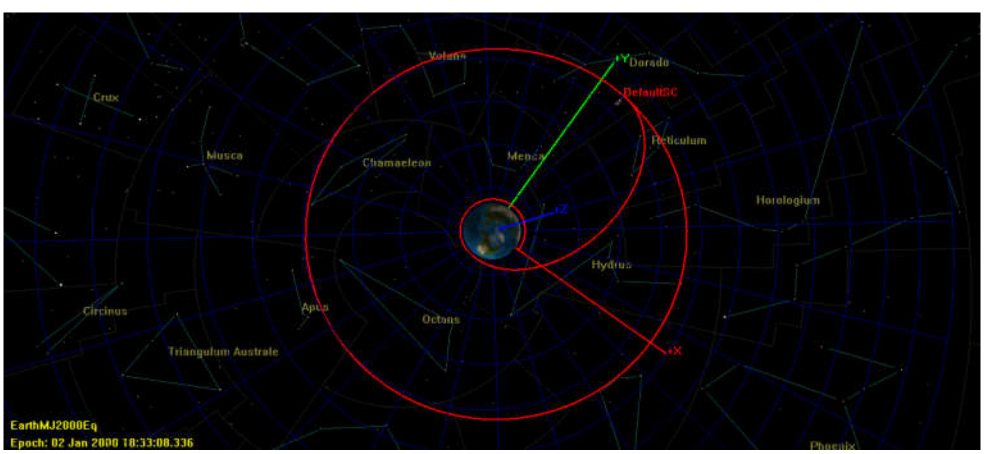

Fig 19: Lunar Transfer **Trajectory as viewed from Moon**

Trajectory as viewed from Moon

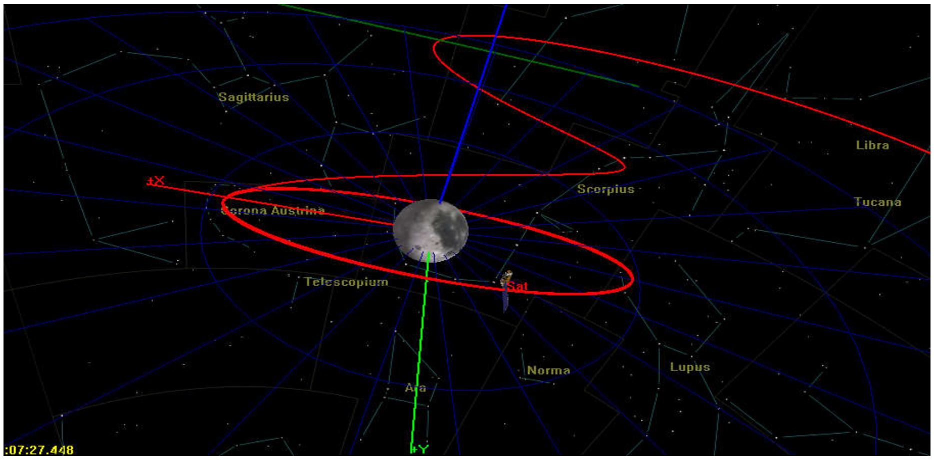

Fig 20: Lunar Transfer from moon

6 CONCLUSION

This Paper is mainly focus how to maintains the orientation of satellite in low earth orbit (LEO) from perturbation also reduce the **Orbital decay** and Increase the **life time** of the satellite and various perturbations like lunar gravity and atmospheric drag will be considered and will be compared with the ideal cases. This perturbation force is affecting the satellite from the original orbit. Then will affect the keplerian elements. This variation is called by secular variation might be less than orbital period or greater than orbital period. This study states for Low Earth Orbit satellite have more Aerodynamic drag and Gravitational attraction due to Earth. And High earth orbit cause more force due to moon attraction and magnetic effect and flattering of earth.

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