CUBIC DIFFERENCE PRIME LABELLING OF SOME PLANAR GRAPHS

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Abstract

Cubic difference prime labelling of a graph is the labelling of the vertices with {0, 1, 2---------, p-1} and the edges with absolute difference of the cubes of the labels of the incident vertices. The greatest common incidence number of a vertex (*gcin*) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the *gcin* of each vertex of degree greater than one is one, then the graph admits cubic difference prime labelling. Here we identify some planar graphs for cubic difference prime labelling.

***Keywords*— Graph labelling, greatest common incidence number, cubic difference, prime labelling, planar graphs.**

I. INTRODUCTION

All graphs in this paper are simple, finite, connected and undirected. The symbol V (G) and E (G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph. A graph labelling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2], [3] and [4]. Some basic concepts are taken from [1] and [2]. In this paper we investigated cubic difference prime labelling of some planar graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

II. MAIN RESULTS

Definition 2.1 Let G = (V (G), E (G)) be a graph with p vertices and q edges. Define a bijection

f : V(G) → {0,1,2,3,----------------,p-1} by f(v_i) = i−1 , for every i from 1 to p and define a 1-1 mapping f_{cdpl}^* : E(G) → set of natural numbers N by $f_{cdpl}^*(uv)$ = |{f(u)}^{3} – { f(v)}^{3}| The induced function f_{cdpl}^* is said to be cubic difference prime labelling, if for each vertex of degree at least 2, the greatest common incidence number of the labels of the incident edges is 1.

Definition 2.2 A graph which admits cubic difference prime labelling is called a cubic difference prime graph.
Theorem 2.1 Triangular belt TB (α), where $\propto = (\uparrow\uparrow -----, n-1\ times)$ admits cubic difference prime labelling.
Proof: Let G = TB (α) and let v_1, v2, ----------------, v2n are the vertices of G
Here |V (G)| = 2n and |E (G)| = 4n-3
Define a function f : V → {0,1,2,3,----------------,2n-1 } by

f (v_i) = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labelling f, the induced edge labelling f_{cdpl}^* is defined as follows
$f_{cdpl}^*(v_i\ v_{i+1})$ = i3-(i-1)^3, i = 1,2,------------,2n-1
$f_{cdpl}^*(v_{2i-1}\ v_{2i+1})$ = 24i2-24i+8, i = 1,2,------------,n-1
$f_{cdpl}^*(v_{2i}\ v_{2i+2})$ = 24i2+2, i = 1,2,------------,n-1

Clearly f^*_{cdpl} is an injection.

gcin of (v_{i+1}) = gcd of $\{f^*_{cdpl}(v_i\ v_{i+1})\ ,\ f^*_{cdpl}(v_{i+1}\ v_{i+2})\ \}$
= gcd of {3i2-3i+1, $3i^2$+3i+1}
= gcd of {$6i^2$, $3i^2$-3i+1}
= gcd of {$3i^2$, $3i^2$-3i+1}
= gcd of {3i-1, $3i^2$-3i+1}
= gcd of {i, 3i-1}
= gcd of {i, i-1} = 1, i = 1,2,------------,2n-2

gcin of (v_1) = 1.

gcin of (v_{2n}) = gcd of $\{f^*_{cdpl}(v_{2n-1}\ v_{2n})\ ,\ f^*_{cdpl}(v_{2n-2}\ v_{2n})\ \}$
= gcd of {12n2-18n+7, $24n^2$-48n+26}
= gcd of {12n2-18n+7, $12n^2$-24n+13}
= gcd of {6n-6, $12n^2$-24n+13}
= gcd of {6n-6, (6n-6)(2n-2)+1}= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence TB (α), admits cubic difference prime labelling. ■

Example 2.1 Let G = TB (↑↑↑↑)

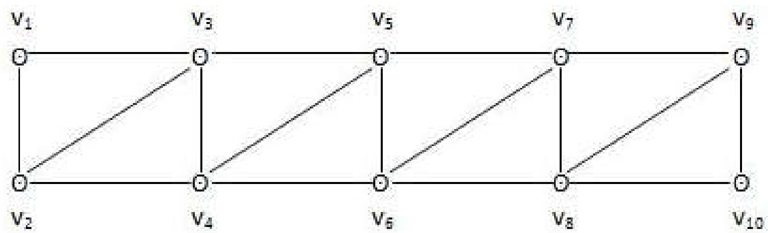

fig – 2.1

Theorem 2.2 Two copies of cycle C_n sharing a common edge admits cubic difference prime labelling.

Proof: Let G = 2(C_n) – e and let v_1, v2, ----------------, v2n-2 are the vertices of G
Here |V (G)| = 2n-2 and |E (G)| = 2n-1
Define a function f : V → {0,1,2,3,----------------,2n-3 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n-2
Clearly f is a bijection.
For the vertex labelling f, the induced edge labelling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_i\ v_{i+1})$ = i^3-$(i-1)^3$, i = 1,2,------------,2n-3

$f^*_{cdpl}(v_1\ v_{2n-2})$ = $(2n-3)^3$.

Case(i) n is even.

$f^*_{cdpl}(v_{\frac{n}{2}}\ v_{(\frac{3n-2}{2})})$ = $26n^3$-$102n^2$+132n-56,

Case(ii) n is odd.

$f^*_{cdpl}(v_{(\frac{n+1}{2})}\ v_{(\frac{3n-1}{2})})$ = $\frac{13}{4}(n-1)^3$.

Clearly f^*_{cdpl} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,2n-4

gcin of (v_1) = gcd of $\{f^*_{cdpl}(v_1\ v_2)\ ,\ f^*_{cdpl}(v_{2n-2}\ v_1)\ \}$
= gcd of {1, $(2n-3)^3$}= 1.

gcin of (v_{2n-2}) = gcd of $\{f^*_{cdpl}(v_1\ v_{2n-2})\ ,\ f^*_{cdpl}(v_{2n-2}\ v_{2n-3})\ \}$
= gcd of { $(2n-3)^3$, $12n^2$-42n+37}
= gcd of { (2n-3), $12n^2$-42n+37}
= gcd of { (2n-3), (2n-3(6n-12)+1}= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence 2(C_n) – e, admits cubic difference prime labelling. ■

Example 2.2 Let G = 2(C_5)-e

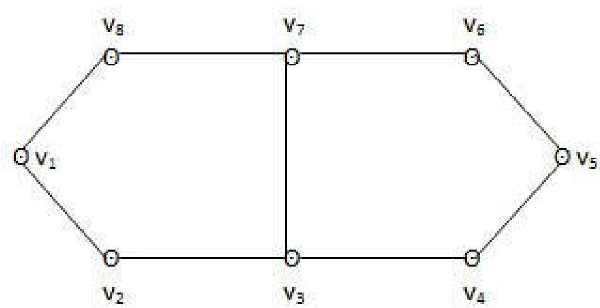

fig – 2.2

Theorem 2.3 The graph P_n (+) N_m admits cubic difference prime labelling.
Proof: Let G = P_n (+) N_m and let v_1, v2, ----------------,v_{n+m} are the vertices of G
Here |V (G)| = n+m and |E (G)| = n+2m-1
Define a function f : V → {0,1,2,3,----------------,n+m-1 } by
$f(v_i) = i-1$, i = 1,2,------,n
$f(u_i) = n+i-1$, i = 1,2,------,m

Clearly f is a bijection.
For the vertex labelling f, the induced edge labelling f^*_{cdpl} is defined as follows
$f^*_{cdpl}(v_{i+1}\ v_{i+2})$ = $i^3-(i-1)^3$, i = 1,2,------------,n-2
$f^*_{cdpl}(v_1\ v_n)$ = $(n-1)^3$.
$f^*_{cdpl}(v_1\ u_i)$ = $(n+i-1)^3$, i = 1,2,------------,m
$f^*_{cdpl}(v_2\ u_i)$ = $(n+i-1)^3-1$, i = 1,2,------------,m
Clearly f^*_{cdpl} is an injection.
gcin of (v_{i+2}) = gcd of {$f^*_{cdpl}(v_{i+2}\ v_{i+3})$, $f^*_{cdpl}(v_{i+1}\ v_{i+2})$ }
= gcd of { $3i^2+9i+7$, $3i^2+3i+1$}
= gcd of {6i+6 , $3i^2+3i+1$}
= gcd of {(i+1) , $3i^2+3i+1$}
= gcd of {(i+1) , 3i(i+1)+1}
= 1, i = 1,2,------------,n-3
gcin of (v_1) = 1.
gcin of (v_2) = 1.
gcin of (v_n) = gcd of {$f^*_{cdpl}(v_1\ v_n)$, $f^*_{cdpl}(v_{n-1}\ v_n)$ }
= gcd of { $(n-1)^3$, $3n^2-9n+7$}
= gcd of { (n-1), $3n^2-9n+7$}
= gcd of { (n-1), (n-1)(3n-6)+1}= 1.
gcin of (u_i) = gcd of {$f^*_{cdpl}(v_1\ u_i)$, $f^*_{cdpl}(v_2\ u_i)$ }
= gcd of { $(n+i-1)^3$, $(n+i-1)^3-1$}
= 1, i = 1,2,------------,m
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence P_n (+) N_m, admits cubic difference prime labelling. ■
Example 2.3 Let G = P_5 (+) N_4

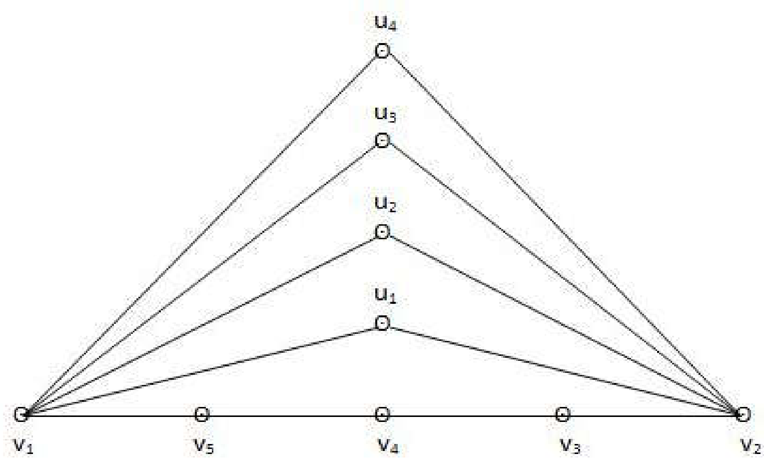

Fig – 2.3
Theorem 2.4 The Jewel graph J_n admits cubic difference prime labelling.
Proof: Let G = J_n and let u, v, x, y, v1, v2, ----------------,v_n are the vertices of G
Here |V (G)| = n+4 and |E (G)| = 2n+5
Define a function f : V → {0,1,2,3,----------------,n+3 } by
$f(v_i) = i+3$, i = 1,2,------,n
f (u) = 0, f(v) = 1, f(x) = 2, f(y) = 3

Clearly f is a bijection.
For the vertex labelling f, the induced edge labelling f^*_{cdpl} is defined as follows
$f^*_{cdpl}(ux)$ = 8, $f^*_{cdpl}(vx)$ = 7
$f^*_{cdpl}(uy)$ = 27, $f^*_{cdpl}(vy)$ = 26, $f^*_{cdpl}(xy)$ = 19
$f^*_{cdpl}(u\ v_i)$ = $(i+3)^3$, i = 1,2,------------,n
$f^*_{cdpl}(v\ v_i)$ = $(i+3)^3-1$, i = 1,2,------------,n
Clearly f^*_{cdpl} is an injection.

gcin of (u) = 1, ***gcin*** of (v) = 1, ***gcin*** of (x) = 1, ***gcin*** of (y) = 1

gcin of (v_i) = gcd of {$f^*_{cdpl}(u\ v_i)$, $f^*_{cdpl}(v\ v_i)$ }
= gcd of {$(i+3)^3$, $(i+3)^3$-1}
= 1, i = 1,2,------------,n

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence J_n, admits cubic difference prime labelling. ■

Example 2.4 Let G = J_3

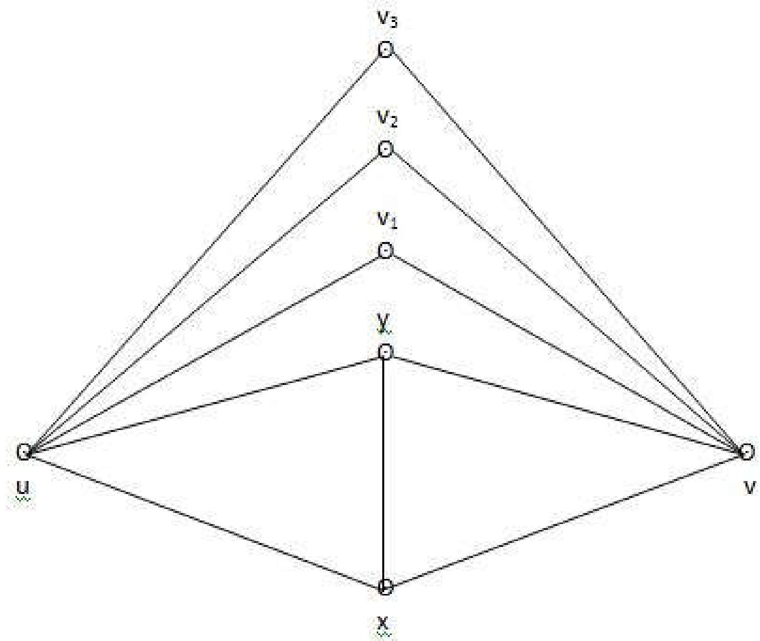

Fig – 2.4

Theorem 2.5 Two copies of cycle C_n sharing a common vertex admits cubic difference prime labelling.

Proof: Let G = 2(C_n) – v and let v_1, v2, ----------------, v2n_{-1} are the vertices of G

Here |V (G)| = 2n-1 and |E (G)| = 2n

Define a function f : V → {0,1,2,3,----------------,2n-2 } by

f(v_i) = i-1 , i = 1,2,------,2n-1

Clearly f is a bijection.

For the vertex labelling f, the induced edge labelling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_i\ v_{i+1})$ = i3-$(i-1)^3$, i = 1,2,------------,n-1

$f^*_{cdpl}(v_{n+i+1}\ v_{n+i})$ = $(n+i)^3$ – $(n+i-1)^3$, i = 1,2,------------,n-2

$f^*_{cdpl}(v_1\ v_n)$ = $(n-1)^3$.

$f^*_{cdpl}(v_1\ v_{2n-1})$ = $(2n-2)^3$.

$f^*_{cdpl}(v_1\ v_{n+1})$ = n3.

Clearly f^*_{cdpl} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,n-2

gcin of (v_1) = gcd of {$f^*_{cdpl}(v_1\ v_2)$, $f^*_{cdpl}(v_n\ v_1)$ }
=gcd of { 1, $(n-1)^3$}= 1.

gcin of (v_{2n-1}) = gcd of {$f^*_{cdpl}(v_1\ v_{2n-1})$, $f^*_{cdpl}(v_{2n-2}\ v_{2n-1})$ }
= gcd of { $(2n-2)^3$, $12n^2$-30n+19}
= gcd of { (2n-2), $12n^2$-30n+19}
= gcd of { (2n-2), (2n-2)(6n-9)+1}= 1.

gcin of (v_n) = gcd of {$f^*_{cdpl}(v_{n-1}\ v_n)$, $f^*_{cdpl}(v_1\ v_n)$ }
= gcd of { $(n-1)^3$, $3n^2$-9n+7}
= gcd of { (n-1), $3n^2$-9n+7}
= gcd of { (n-1), (n-1)(3n-6)}= 1.

gcin of (v_{n+1}) = gcd of {$f^*_{cdpl}(v_{n+1}\ v_{n+2})$, $f^*_{cdpl}(v_1\ v_{n+1})$ }
= gcd of { n^3, $3n^2$+3n+1}
= gcd of { n, $3n^2$+3n+1} = 1

gcin of (v_{n+i+1}) = 1, i = 1,2,------------,n-3

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence 2(C_n) – v, admits cubic difference prime labelling. ■

Example 2.5 Let G = 2(C_5) –v

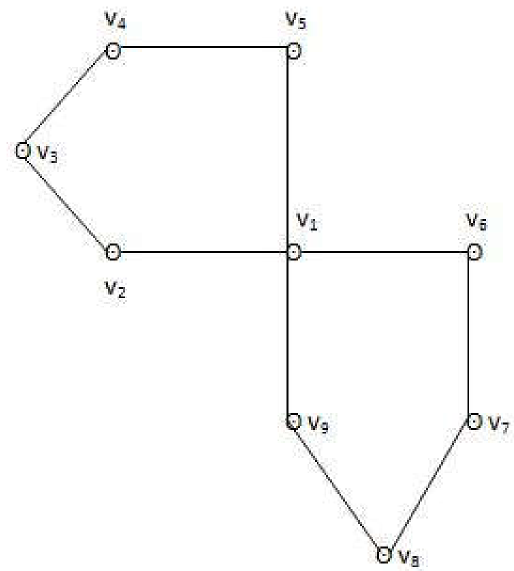

Fig -2.5

Theorem 2.6 The graph PL_2 (n) admits cubic difference prime labelling.

Proof: Let G = PL_2 (n) and let a,b, v_1,v_2,----------------,v_n are the vertices of G

Here |V (G)| = n+2 and |E (G)| = 3n

Define a function f : V → {0,1,2,3,----------------,n+1 } by

$f(v_i) = i+1$, i = 1,2,------,n

f(a) = 0 , f(b) = 1.

Clearly f is a bijection.

For the vertex labelling f, the induced edge labelling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_i\, v_{i+1})$ = 3i2+9i+7, i = 1, 2, ------------, n-1

$f^*_{cdpl}(a\, v_i)$ = $(i+1)^3$, i = 1, 2, ------------, n

$f^*_{cdpl}(b\, v_i)$ = $(i+1)^3-1$, i = 1, 2, ------------, n

$f^*_{cdpl}(ab)$ = 1.

Clearly f^*_{cdpl} is an injection.

gcin of (v_{i+2}) = 1, i = 1,2,------------,n-3

gcin of (a) = 1.

gcin of (b) = 1.

gcin of (v_i) = gcd of {$f^*_{cdpl}(a\, v_i)$, $f^*_{cdpl}(b\, v_i)$ }

= gcd of {$(i+1)^3$, $(i+1)^3-1$}

= 1, i = 1,2,------------,n

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence PL_2 (n), admits cubic difference prime labelling. ■

Example 2.6 Let G = PL_2 (4)

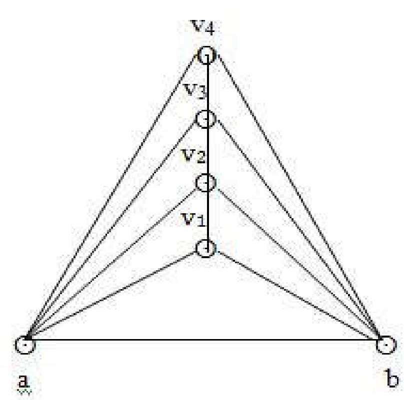

Fig -2.6

Theorem 2.7 Jelly fish graph JFn admits cube difference prime labelling.
Proof: Let G = JF_n and let a, b, x, y, v1, v2, ----------------, vn, u_1, u2, ----------------, un are the vertices of G
Here |V (G)| = 2n+4 and |E (G)| = 2n+5
Define a function f : V → {0,1,2,3,----------------,2n+3 } by

f (v_i) = n+ i+3 , i = 1,2,------,n

f (u_i) = i+3 , i = 1,2,------,n

f (a) = 0, f (b) = 1, f(x) = 2, f(y) = 3

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{sqsp}(ay)$ = 27
$f^*_{sqsp}(by)$ = 26
$f^*_{sqsp}(ax)$ = 8
$f^*_{sqsp}(bx)$ = 7
$f^*_{sqsp}(xy)$ = 19
$f^*_{sqsp}(a\ u_i)$ = $(i+3)^3$, 1≤ i ≤ n.
$f^*_{sqsp}(b\ v_i)$ = $(n+i+3)^3$ -1, 1≤ i ≤ n.
Clearly f^*_{cdpl} is an injection.
gcin of (a) = 1
gcin of (b) = 1
gcin of (x) = 1
gcin of (y) = 1
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence JF_n, admits cubic difference prime labelling. ■

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