FORCED CONVECTIVE HEAT FLOW OF A LIQUID FOR DIFFERENT DEPTHS OF THE CHANNEL WITH A CONSTANT HEAT SOURCE

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Abstract: **The present paper analyses a steady flow of a viscous fluid of finite depth in a porous medium over a fixed horizontal, impermeable bottom with a uniformly distributed constant heat source in the flow region. The governing equations are solved analytically when the temperatures on the fixed bottom and on the free surface are prescribed. The effects of pertinent parameters on velocity profiles, temperature profiles and skin friction are shown graphically.**

Keywords: Forced Convection, Channel, Heat Source

I. INTRODUCTION

The newer results developed for heat transfer through porous media refer to forced convection has received increasing interest over the last 45 years, due its numerous applications in geophysics and energy-related systems. Convection in ducted flow may occur in many applications, such as heat exchangers, chemical processing equipment, transport of heated or cooled fluids, solar collectors, and micro-electronic cooling. Buoyancy effects distort the velocity and temperature profiles relative to the forced convection case. This phenomenon is of substantial significance because it may strongly affect wall friction, pressure drop, heat transfer, heat exchangers simulation and design, occurrence of extreme temperatures and stability of the flow, tree networks of cracks (construal theory), time-dependent heating, annular channels, stepwise changes in wall temperature, local thermal nonequilibrium, and other external flows (such as over a cone or wedge), fluid inertia, thermal dispersion, boundary friction, non-Newtonian fluids, and porosity variation. In view of the above some authors studied Vafai and Tien [1] considered Boundary and inertia effects on flow and heat transfer in porous media, Vargas et.al [2] analyzed Nonsimilar solution for mixed convection on a wedge embedded in a porous medium, Bejan et.al [3] Porous and complex fluid structures in modern technologies, Carslaw and Jaeger [4] Conduction of heat in solids, 2nd ed, Ingham and Pop [5] Transport phenomena in porous media II, Peterscu [6] Comments on the optimal spacing of parallel plates cooled by forced convection, Trevisan and Bejan [7] Mass and heat transfer by natural convection in a vertical slot filled with porous medium. Yasir Ali and Arshad Alam Khan [8] studied exact solution of magnetohydrodynamic slip flow and heat transfer over an oscillating and translating porous plate, Yama Moto and Jwamura [9] analysed flow with convective acceleration through a porous medium. Ch Kesavaiah et.al. [10] Studied radiation absorption, chemical reaction and magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux, Ch Kesavaiah et.al. [11] Considered effects of radiation and free convection currents on unsteady Couette flow between two vertical parallel plates with constant heat flux and heat source through porous medium, Chenna Kesavaiah and Venkateswarlu [12] Chemical reaction and radiation absorption effects on convective flows past a porous vertical wavy channel with travelling thermal waves, Chenna Kesavaiah et.al. [13] MHD rotating fluid past a moving vertical plate in the presence of chemical reaction.

II. FORMULATION OF THE PROBLEM

Consider the steady forced convective flow of a Newtonian fluid through a porous medium of viscosity coefficient μ and of finite depth (H) over a fixed horizontal impermeable bottom. The flow is generated by a constant pressure gradient parallel to the plate. Further the bottom is kept at a constant temperature (T_0) and the free surface is exposed to atmospheric temperature (T_1) with uniformly distributed constant heat source (Q) in the flow region. With reference to a rectangular Cartesian coordinates system with the origin on the bottom, $X-$axis in the flow direction, axis vertically upwards and the bottom is represented as $Y=0$ and the free surface as $Y=H$

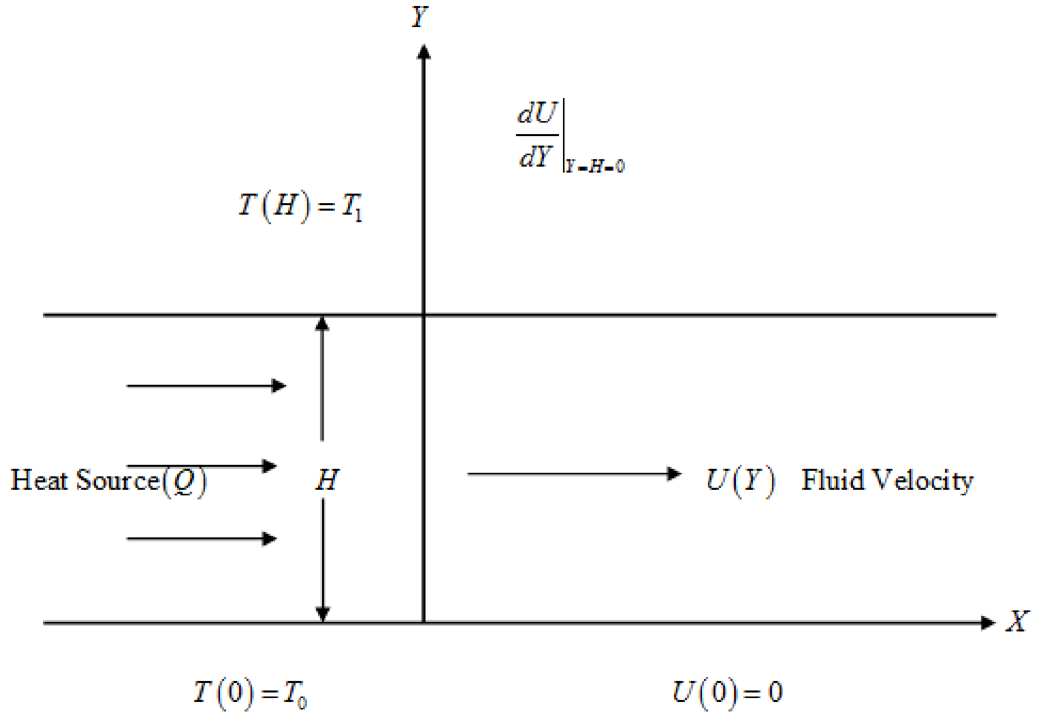

Figure (1): Physical model and coordinates system

Let the convective flow by characterized by the velocity field $u=(U(Y),0,0)$ and the temperature $T(Y)$. This choice of the velocity satisfied.

Continuity equation:

$$\frac{\partial U}{\partial X}=0 \tag{1}$$

Momentum equation:

$$-\frac{\partial P}{\partial X}+\mu\frac{\partial^2 U}{\partial Y^2}-\mu\frac{U}{k^*}=0 \tag{2}$$

Energy equation:

$$\rho c_p U\frac{\partial T}{\partial X}=\kappa\frac{\partial^2 T}{\partial Y^2}+Q \tag{3}$$

where Q is a constant heat source distributed uniformly in the flow region. In the above equations ρ is the fluid density, k^* is the coefficient of porosity of the medium, c_p is the specific heat, K is the thermal conductivity of the fluid and P is the fluid pressure.

The boundary conditions are:

$$U=0,\qquad T=T_0\qquad at\ Y=0$$
$$\mu\frac{dU}{dY}=0,\qquad T=T_1\qquad at\ \ Y=H \tag{4}$$

on introducing the following non – dimensional quantities

$$Y = ay,\ X = ax,\ \ H = ah,\ U = \frac{\mu u}{\rho a^2},\ \ P = \frac{\mu^2 p}{\rho a^2},\ \ T = T_0 + \left(T_1 - T_0\right),\ \Pr = \frac{\mu c_p}{\kappa}$$

$$k^* = \frac{a^2}{\alpha^2}, -\frac{\partial P}{\partial X} = \frac{\mu^2 l_1}{\rho a^3}\left(\because l_1 = \frac{\partial p}{\partial x}\right), \frac{\partial T}{\partial X} = \frac{T_1 - T_0}{a} l_2 \left(\because l_2 = \frac{\partial \theta}{\partial x}\right),\ f = \frac{a^2 Q}{K\left(T_1 - T\right)_0} \qquad (5)$$

where a is some standard length, in view of the above non-dimensional quantities the basic field equations (2) and (3) can be written as:

$$\frac{d^2 u}{dy^2} - \alpha^2 u = -l_1 \qquad (6)$$

$$\frac{d^2 \theta}{dy^2} = \Pr l_2 u - f \qquad (7)$$

The corresponding boundary conditions (4) become

$$u = 0, \qquad \theta = 0 \qquad at\ \ y = 0$$

$$\frac{du}{dy} = 0, \qquad \theta = 1 \qquad at\ \ y = h \qquad (8)$$

III. METHOD OF SOLUTION

Equation (6) and (7) are the first order ordinary differential equations are solving together with boundary conditions (8) we get the following form:

$$u\left(y\right) = K_1 + K_2 e^{-\alpha y} + K_3 e^{\alpha y}$$

The flow rate in the non-dimensional form is

$$q = \int_0^h u\left(y\right) dy = K_1 h + \frac{K_2}{\alpha}\left(e^{-\alpha h} - 1\right) + \frac{K_3}{\alpha}\left(e^{\alpha h} - 1\right)$$

The mean velocity in the non-dimensional form is

$$\frac{1}{h}\int_0^h u\left(y\right) dy = \frac{1}{h}\left[K_1 h + \frac{K_2}{\alpha}\left(e^{-\alpha h} - 1\right) + \frac{K_3}{\alpha}\left(e^{\alpha h} - 1\right)\right]$$

The temperature distribution

$$\theta\left(y\right) = Z_1 y^2 + Z_2 e^{-\alpha y} + Z_3 e^{\alpha y} + Z_4 y^2 + Z_5 + Z_7 y$$

The mean temperature in non-dimensional form is given by

$$\bar{\theta} = \frac{1}{h}\int_0^h \theta\, dy = \frac{1}{h}\int_0^h \left(Z_1 y^2 + Z_2 e^{-\alpha y} + Z_3 e^{\alpha y} + Z_4 y^2 + Z_5 + Z_7 y\right) dy$$

$$= \frac{1}{h}\left[\frac{Z_1}{3} h^3 - \frac{Z_2}{\alpha} e^{-\alpha h} + \frac{Z_3}{\alpha} e^{\alpha h} - \frac{Z_4}{3} h^3 - Z_5 h + \frac{Z_7}{2} h^2 + \frac{Z_2}{\alpha} - \frac{Z_3}{\alpha}\right]$$

Mean mixed temperature in the dimensionless form:

$$\frac{\int_0^h \theta\left(y\right) dy}{\int_0^h u\left(y\right) dy} = \frac{\frac{Z_1}{3} h^3 - \frac{Z_2}{\alpha} e^{-\alpha h} + \frac{Z_3}{\alpha} e^{\alpha h} - \frac{Z_4}{3} h^3 - Z_5 h + \frac{Z_7}{2} h^2 + \frac{Z_2}{\alpha} - \frac{Z_3}{\alpha}}{K_1 h + \frac{K_2}{\alpha}\left(e^{-\alpha h} - 1\right) + \frac{K_3}{\alpha}\left(e^{\alpha h} - 1\right)}$$

Coefficient of Skin-Friction

The coefficient of skin-friction at the vertical porous surface is given by

On the bottom:

$$\tau = \left(\frac{du}{dy}\right)_{y=0} = -\alpha K_2 + \alpha K_3$$

On the Surface:

$$\tau = \left(\frac{du}{dy}\right)_{y=h} = -\alpha K_2 e^{-\alpha h} + \alpha K_3 e^{\alpha h}$$

Coefficient of Heat Transfer (Nusselt number):

The rate of heat transfer in terms of Nusselt number at the vertical porous surface is given by

On the bottom:

$$N_u = \left(\frac{d\theta}{dy}\right)_{y=0} = -\alpha Z_2 + \alpha Z_3$$

On the free Surface:

$$N_u = \left(\frac{d\theta}{dy}\right)_{y=h} = -2Z_1 h - \alpha Z_2 e^{-\alpha h} + \alpha Z_3 e^{\alpha h} + 2Z_4 h + Z_7$$

IV. RESULTS AND DISCUSSIONS

The velocity profiles for different values of porosity parameter (α) determined in figure (2), it is clear that the velocity decreases with increasing values of the porosity parameter and also we found that thickness of the boundary layer decreases as the porosity parameter increases for large values of channel wall. It can be observed the variations of velocity profiles depicted for different values of depth of the channel (h) in figure (3), it was found that the velocity increases with increasing values of depth of the channel for thin fluids. Figure (4) shows that the velocity profiles for different values of pressure gradient (l_1), it was observed that the velocity increases with increasing values of pressure gradient. Figure (5) displays the velocity variations for different values of heat source parameter (f), it is pragmatic that the velocity increases with increasing values of heat source parameter. The temperature variations observed from figure (6) for different values of porosity parameter (α), it is readily apparent that the temperature of fluid flow slightly decreases with increasing values of porosity parameter when heat source parameter $f = 10$. Figure (7) and (8) illustrate the temperature profiles for different values of Prandtl number $(\Pr)$ and depth of the channel (h). Form these figures we observed that the temperature profiles downward with increasing values of Prandtl number where are the reverse effect observed for depth of the channel. The temperature profiles for different values of pressure gradient (l_1) and l_2 are predicted in figures (9) and (10). It is clear from these figures that an increasing values of l_1 and l_2 the temperature also increases for both. Skin friction coefficient observed in figure (11) for different values of porosity parameter (α), we found that an increases in porosity parameter the results were decreases.

APPENDIX

$$K_1 = \frac{l_1}{\alpha^2}, K_2 = -\frac{K_1 \alpha e^{\alpha h}}{2\alpha \cosh(\alpha h)}, K_3 = -(K_1 + K_2), Z_1 = \frac{\Pr l_2 K_1}{2}, Z_2 = \frac{\Pr l_2 K_2}{2}, Z_3 = \frac{\Pr l_2 K_3}{2}$$

$$Z_4 = -\frac{f}{2}, Z_5 = -(Z_2 + Z_3), Z_6 = \left(Z_5 + Z_1 h^2 + Z_2 e^{-\alpha h} + Z_3 e^{\alpha h} + Z_4 h^2\right), Z_7 = (1 - Z_6)$$

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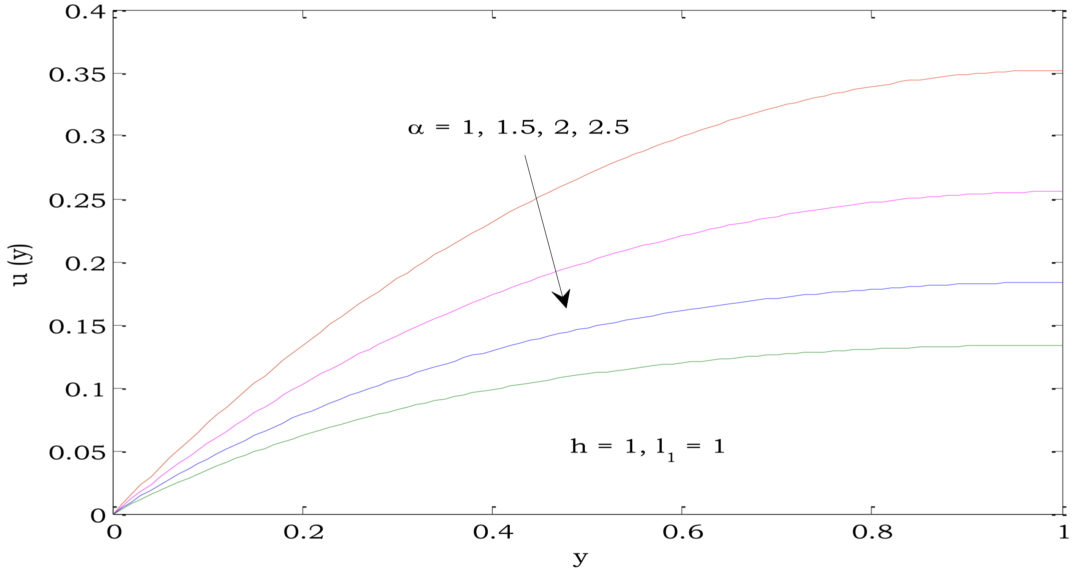

Figure (2): Velocity profiles for different values of α

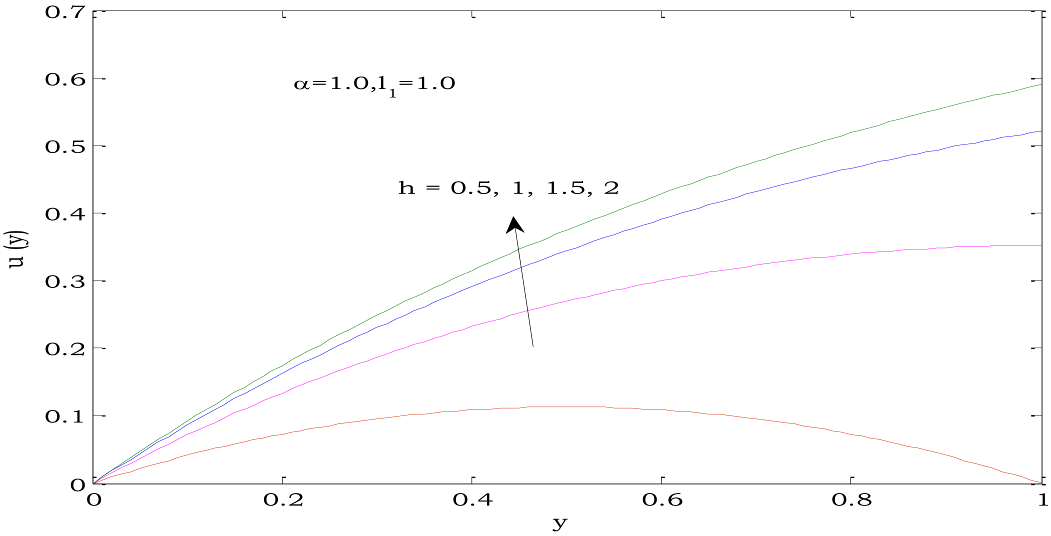

Figure (3): Velocity profiles for different values of h

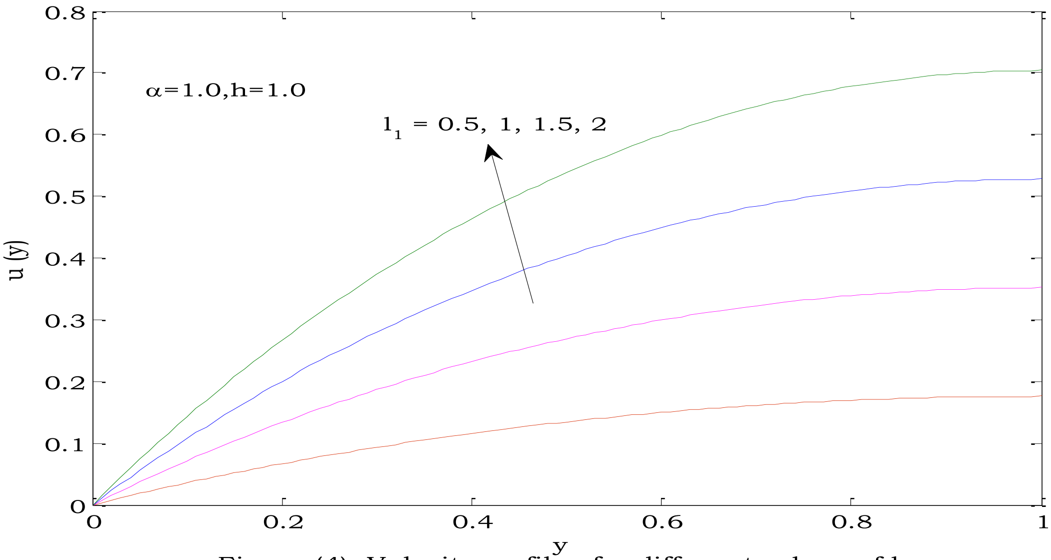

Figure (4): Velocity profiles for different values of l_1

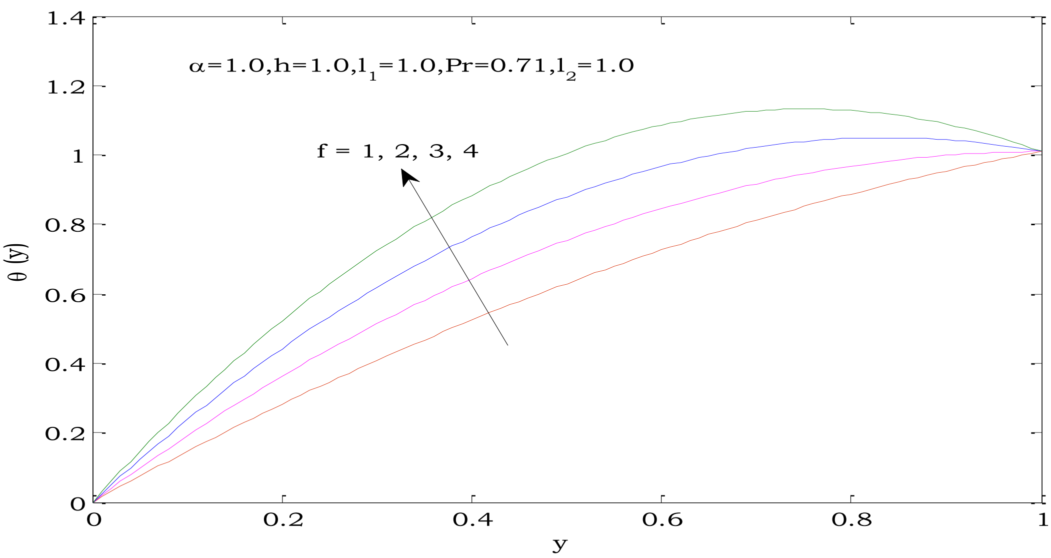

Figure (5): Temperature profiles for different values of f

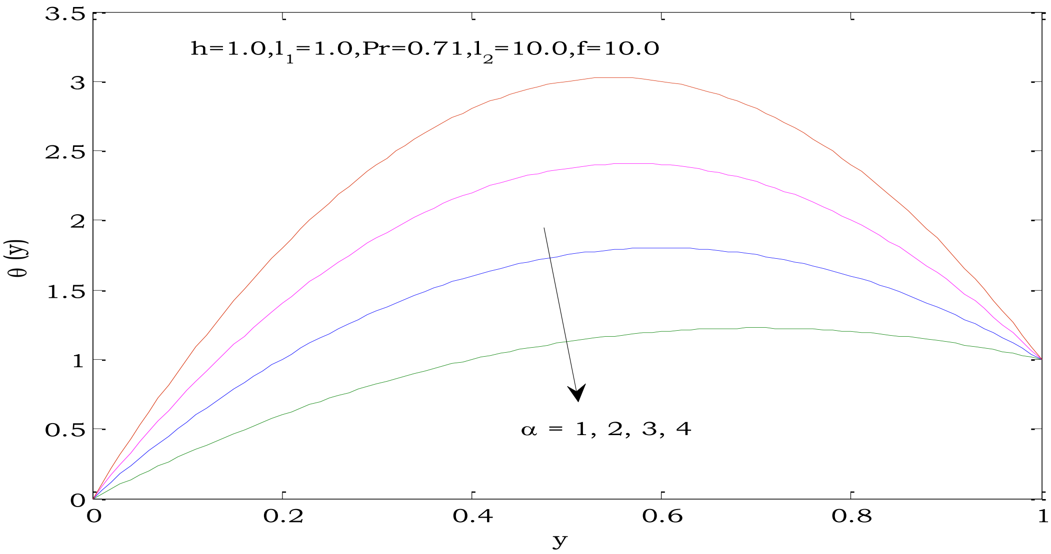

Figure (6): Temperature profiles for different values of α

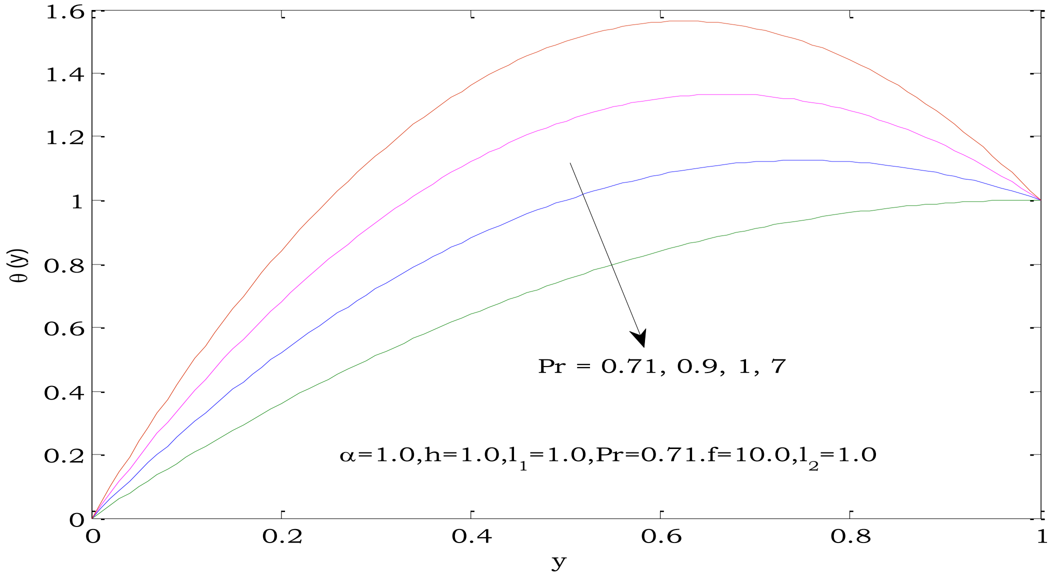

Figure (7): Temperature profiles for different values of Pr

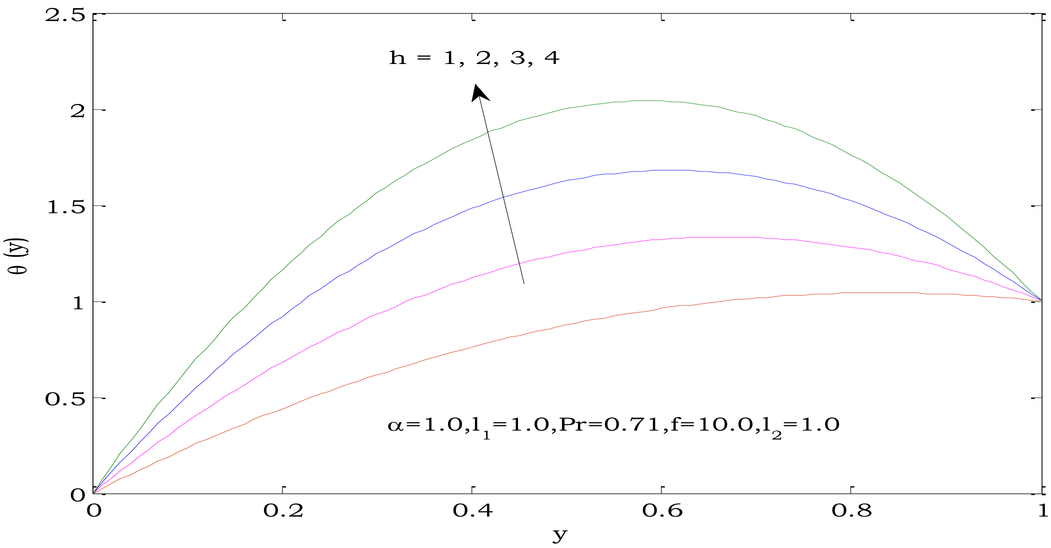

Figure (8): Temperature profiles for different values of h

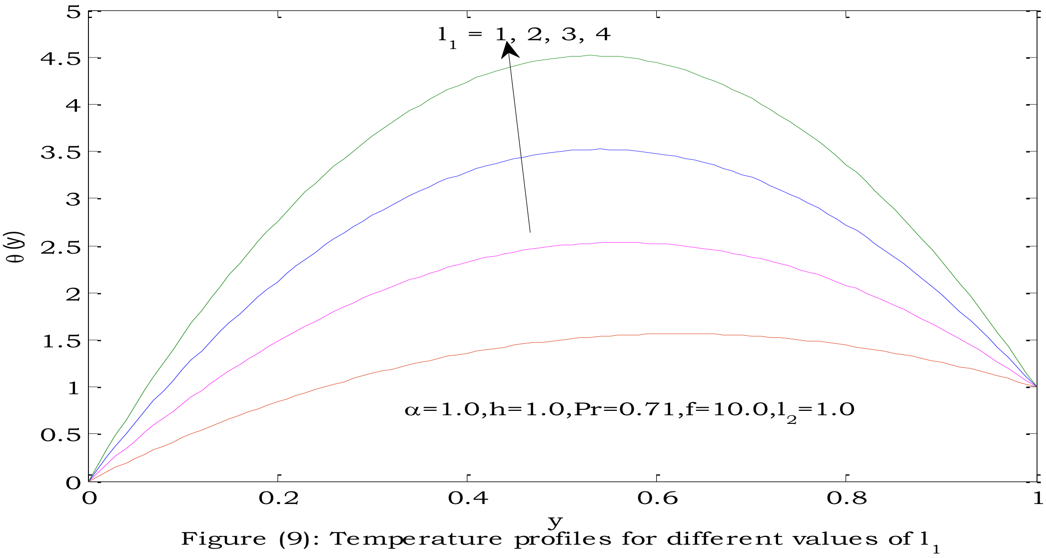

Figure (9): Temperature profiles for different values of l_1

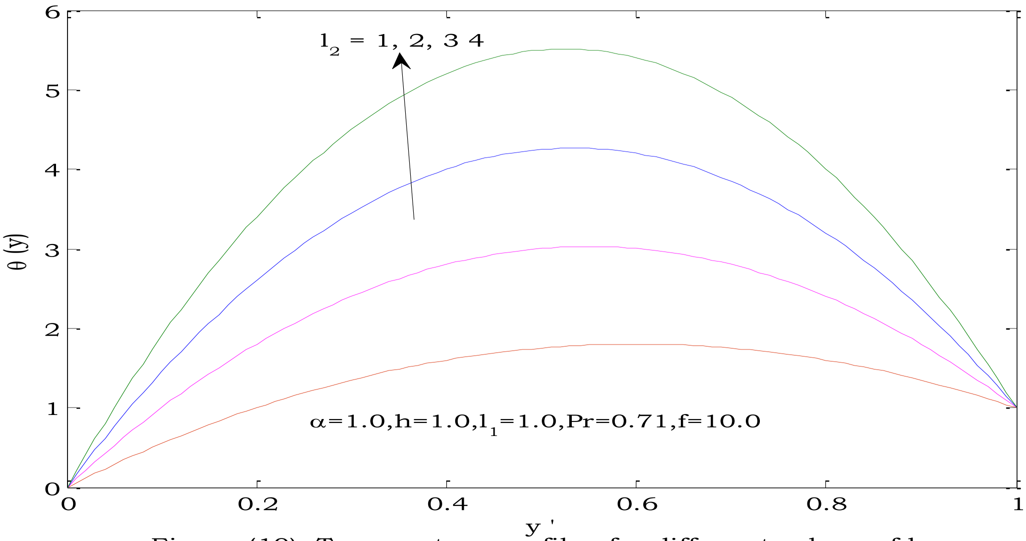

Figure (10): Temperature profiles for different values of l_2

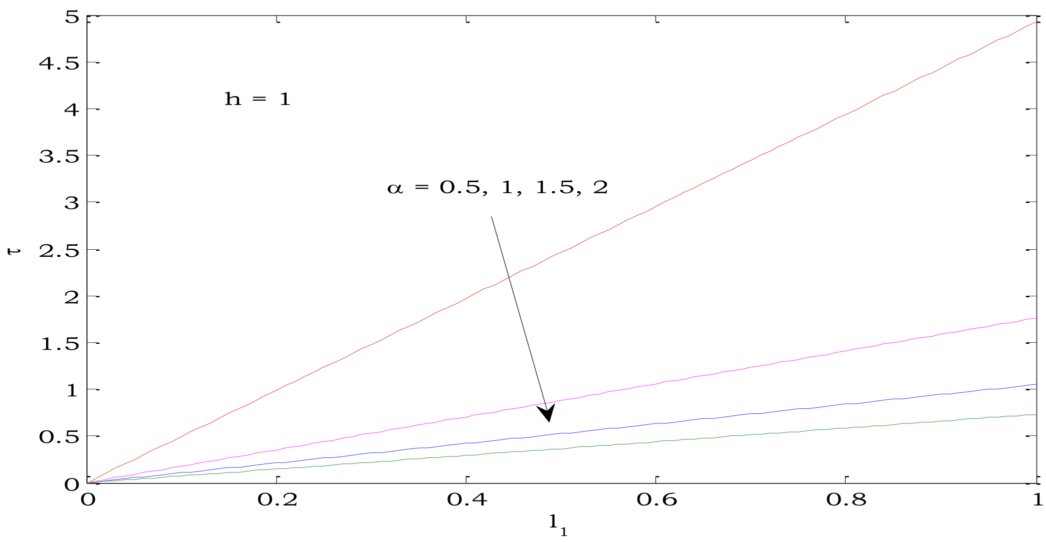

Figure (11): Skin friction for different values of α