FUZZY GRACEFUL LABELING AND FUZZY ANTIMAGIC LABELING OF PATH AND BUTTERFLY GRAPH

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***Abstract* - Fuzzy antimagic labeling introduced in this paper. Examine results in vertex antimagic labeling and edge antimagic labeling for some graphs like path graph and butterfly graph. Fuzzy graceful labelling introduced [9].Properties are defined for path graph and butterfly graph through Fuzzy graceful labelling.**

***Key words* - Fuzzy labelling, Graceful labelling, fuzzy antimagic path graph, fuzzy antimagic butterfly graph.**

***AMS Mathematics subject classification*: 05C,94D**

I. Introduction

A magic square is an arrangement of numbers into a square such that the sum of each row , column and diagonal are equal. The term antimagic then comes from being the opposite of magic or arranging numbers in a way such that no two sums are equal[3]. The interest in graph labelings can trace its roots back to a paper [4] by A.Rosa in 1966. Nora Hartsfield and Gerhard Ringel [5] introduced the concept of antimagic labelling which is an assignment of distinct values to different objects in a graph such a way that when taking certain sums of the labels the sums will all be different.

Bodendiek and Walter [6] defined the concept of an antimagic labeling as an edge labelling in which the vertex values form an arithmetic progression starting from a and have common difference d. Martin Baca, Francois Bertault and MacDougall[7] introduce the notions of the vertex antimagic labelling and Nissankara[8] derived the algorithm for vertex antimagic labelling.

A graceful labelling of a graph G with 'q' edges and vertex set V is an injection $f:V(G)\to\{0,1,2,...q\}$with the property that the resulting edge labels are also distinct, where an edge incident with vertices u and v is assigned the label $|f(u)-f(v)|$. A graph which admits graceful labelling is called graceful graph[1]. Various kinds of graphs are shown to be graceful. In this paper we developed graceful conditions for fuzzy antimagic path graph and butterfly graph.

II. Preliminaries and Main Results

Definition:1

Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of U xV.

Definition: 2

A fuzzy graph $G =(\sigma, \mu)$ is a pair of functions $\sigma : V\to [0,1]$ and $\mu : V\times V \to [0,1]$, where for all $u,v \in V$, we have $\mu (u,v) \leq \sigma(u) \wedge \sigma(v)$.

Definition: 3

A labelling of a graph is an assignment of values to the vertices and edges of a graph.

Definition:4

A graceful labelling of a graph G with q edges is an injection $f: V(G) \to \{0,1,2,\dots,q\}$ such that when each edge $xy \in E(G)$ is assigned the label $|f(x) - f(y)|$, all of the edge labels are distinct.

Definition:5

A graph $G =(\sigma, \mu)$ is said to be a fuzzy graceful labelling graph if $\sigma : V\to [0,1]$ and

μ : VxV → [0,1] is bijective such that the membership value of edges and vertices are distinct and μ (u,v) $\leq$ σ(u) $\wedge$ σ(v) for all u,v ϵ V.

Definition: 6

A Graph G is called antimagic if the n edges of G can be distinctly labelled 1 through n in such a way that when taking the sum of the edge labels incident to each vertex, the sums will all be different.

Definition: 7

A fuzzy labelling graph is said to be a fuzzy magic graph if σ (u)+μ(u,v)+σ(v) has a same value for all u,vϵV which is denoted as m_o(G).

Definition: 8

A fuzzy antimagic graph G is a bijection f:E(G)→{(1,2,3,.....│ E(G)│)z} such that for any two distinct vertices u and v, the sum of the labels on edges incident to u is different from the sum of the labels on edges incident to v.

Definition: 9

In a fuzzy antimagic path graph if σ (u)+μ(u,v)+σ(v) has distinct values for every u,vϵV and each edge uvϵE(G) is assigned the label │σ (u)-σ(v)│for all u,vϵV then it is graceful .

Definition: 10

A butterfly graph B_n is a simple graph connecting two copies of fan graph and two pendent vertices with a common vertex.

Definition: 11

In a fuzzy antimagic butterfly graph if μ (v_i,v_{i+1}) + σ(v_{i+1}) + μ(v_{i+1},v_{i+2}) has distinct values for every v_iϵV and each edge v_i,v_{i+1} ϵE(G) is assigned the label │σ (v_i)-σ(v_{i+1})│for all v_i ϵV then it is graceful.

Theorem: 1

Path graph P_n is fuzzy antimagic labelling graph for every $3 \leq n \leq 17$.

Proof :

Let $v_1,v_2,v_3...v_n$ be the vertices and $e_1,e_2,e_3...e_{n-1}$ be the edges of the path graph P_n.

Let z→(0,1] such that one can choose z = 0.01 if n ≤ 17 and z= 0.001 if n > 17

Consider the following cases.

Case(i): $\sigma(v_i)$=2iz

Path graph P_n is a fuzzy antimagic graph for every 3≤n≤17 when $\sigma(v_i)$=2iz, where i=1,2,...17and z=0.01.

In these graphs,

σ (v_1) =2*(1z) =0.02

σ (v_2) =2*(2z) =0.04

σ (v_3) =2*(3z)=0.06

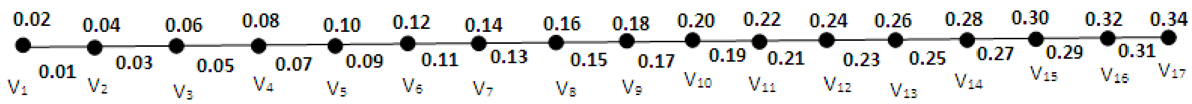
....

σ (v_i) =2*(iz), for every i=1,2,...17

σ (v_{i+1}) –σ (v_i) =2z , for every i=1,2,...16.

Also σ (v_i)+μ(v_iv_{i+1})+σ(v_{i+1})≤1 and are distinct to each other.

Example:



Fuzzy antimagic path graph – P_{17}

Case(ii): $\sigma(v_i)$=3iz

Path graph P_n is a fuzzy antimagic graph for every 3≤n≤17 when $\sigma(v_i)$=3iz, where i=1,2,...17and z=0.01.

In these graphs,

σ (v_1) =3*(1z) =0.03

σ (v_2) =3*(2z) =0.06

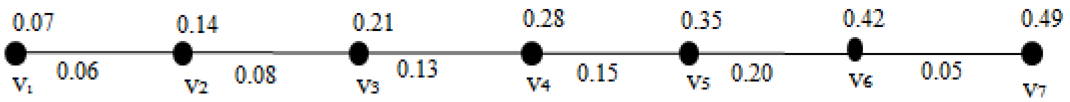
σ (v_3) =3*(3z) =0.09

....

σ (v_i) =3*(iz), for every i=1,2,...17

$\sigma(v_{i+1}) - \sigma(v_i) = 3z$, for every i=1,2,...16.
Also $\sigma(v_i)+\mu(v_iv_{i+1})+\sigma(v_{i+1})\leq 1$ and are distinct to each other.

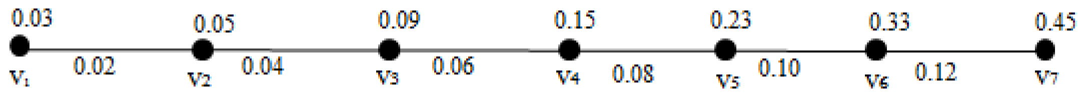
Case(iii): $\sigma(v_i)=4iz$
Path graph P_n is a fuzzy antimagic graph for every $3\leq n\leq 12$ when $\sigma(v_i)=4iz$, where i=1,2,...12and z=0.01.
In these graphs,
$\sigma(v_1) = 4*(1z) = 0.04$
$\sigma(v_2) = 4*(2z) = 0.08$
$\sigma(v_3) = 4*(3z) = 0.12$
....
$\sigma(v_i) = 4*(iz)$, for every i=1,2,...12
$\sigma(v_{i+1}) - \sigma(v_i) = 4z$, for every i=1,2,...11.
Also $\sigma(v_i)+\mu(v_iv_{i+1})+\sigma(v_{i+1})\leq 1$ and are distinct to each other.
Case(iv): $\sigma(v_i)=5iz$
Path graph P_n is a fuzzy antimagic graph for every $3\leq n\leq 10$ when $\sigma(v_i)=5iz$, where i=1,2,...10 and z=0.01.
In these graphs,
$\sigma(v_1) = 5*(1z) = 0.05$
$\sigma(v_2) = 5*(2z) = 0.10$
$\sigma(v_3) = 5*(3z) = 0.15$
....
$\sigma(v_i) = 5*(iz)$, for every i=1,2,...10
$\sigma(v_{i+1}) - \sigma(v_i) = 5z$, for every i=1,2,...9.
Also $\sigma(v_i)+\mu(v_iv_{i+1})+\sigma(v_{i+1})\leq 1$ and are distinct to each other.
Case(v): $\sigma(v_i)=6iz$
Path graph P_n is a fuzzy antimagic graph for every $3\leq n\leq 8$ when $\sigma(v_i)=6iz$, where i=1,2,...8 and z=0.01.
In these graphs,
$\sigma(v_1) = 6*(1z) = 0.06$
$\sigma(v_2) = 6*(2z) = 0.12$
$\sigma(v_3) = 6*(3z) = 0.18$
....
$\sigma(v_i) = 6*(iz)$, for every i=1,2,...8
$\sigma(v_{i+1}) - \sigma(v_i) = 2z$, for every i=1,2,...7.
Also $\sigma(v_i)+\mu(v_iv_{i+1})+\sigma(v_{i+1})\leq 1$ and are distinct to each other.
Case(vi): $\sigma(v_i)=7iz$
Path graph P_n is a fuzzy antimagic graph for every $3\leq n\leq 7$ when $\sigma(v_i)=7iz$, where i=1,2,...7and z=0.01.
In these graphs,
$\sigma(v_1) = 7*(1z) = 0.07$
$\sigma(v_2) = 7*(2z) = 0.14$
$\sigma(v_3) = 7*(3z) = 0.21$
....
$\sigma(v_i) = 7*(iz)$, for every i=1,2,...7
$\sigma(v_{i+1}) - \sigma(v_i) = 7z$, for every i=1,2,...6.
Also $\sigma(v_i)+\mu(v_iv_{i+1})+\sigma(v_{i+1})\leq 1$ and are distinct to each other.
Example:



Fuzzy antimagic path graph – P_7
Case(vii): $\sigma(v_i)=8iz$ and $\sigma(v_i)=9iz$
Path graph P_n is a fuzzy antimagic graph for every $3\leq n\leq 6$ when $\sigma(v_i)=8iz$ and $\sigma(v_i)=9iz$, where i=1,2,...6 and z=0.01.
Here
$\sigma(v_i) = 8*(iz)$, for every i=1,2,...6
$\sigma(v_{i+1}) - \sigma(v_i) = 8z$, for every i=1,2,...5.
As well as
$\sigma(v_i) = 9*(iz)$, for every i=1,2,...6

$\sigma(v_{i+1}) - \sigma(v_i) = 9z$, for every i=1,2,...5.
Also $\sigma(v_i)+\mu(v_iv_{i+1})+\sigma(v_{i+1})\leq 1$ and are distinct to each other.
Case(viii): $\sigma(v_i)=10iz$ and $\sigma(v_i)=11iz$
Path graph P_n is a fuzzy antimagic graph for every $3\leq n\leq 5$ when $\sigma(v_i)=10iz$ and $\sigma(v_i)=11iz$, where i=1,2,...5 and z=0.01.
Here
$\sigma(v_i) = 10*(iz)$, for every i=1,2,...5
$\sigma(v_{i+1}) - \sigma(v_i) = 10z$, for every i=1,2,...4.
As well as
$\sigma(v_i) = 11*(iz)$, for every i=1,2,...5
$\sigma(v_{i+1}) - \sigma(v_i) = 11z$, for every i=1,2,...4.
Also $\sigma(v_i)+\mu(v_iv_{i+1})+\sigma(v_{i+1})\leq 1$ and are distinct to each other.
Case(ix): $\sigma(v_i)=12iz$, $\sigma(v_i)=13iz$ and $\sigma(v_i)=14iz$
Path graph P_n is a fuzzy antimagic graph for every $3\leq n\leq 4$ when $\sigma(v_i)=12iz$, $\sigma(v_i)=13$ and $\sigma(v_i)=14iz$, where i=1,2,...4 and z=0.01.
Here
$\sigma(v_i) = 12*(iz)$, for every i=1,2,...4
$\sigma(v_{i+1}) - \sigma(v_i) = 12z$, for every i=1,2,3
As well as
$\sigma(v_i) = 13*(iz)$, for every i=1,2,...4
$\sigma(v_{i+1}) - \sigma(v_i) = 13z$, for every i=1,2,3.
and
$\sigma(v_i) = 14*(iz)$, for every i=1,2,...4
$\sigma(v_{i+1}) - \sigma(v_i) = 14z$, for every i=1,2,3.
Also $\sigma(v_i)+\mu(v_iv_{i+1})+\sigma(v_{i+1})\leq 1$ and are distinct to each other.
Case(x): $\sigma(v_i)=15iz$, $\sigma(v_i)=16iz$, $\sigma(v_i)=17iz$, $\sigma(v_i)=18iz$ and $\sigma(v_i)=19iz$
Path graph P_n is a fuzzy antimagic graph for n=3 when $\sigma(v_i)=15iz$, $\sigma(v_i)=16iz$, $\sigma(v_i)=17iz$, $\sigma(v_i)=18iz$ and $\sigma(v_i)=19iz$, where i=1,2,3 and z=0.01.
Here
$\sigma(v_i) = 15*(iz)$, for every i=1,2,3
$\sigma(v_{i+1}) - \sigma(v_i) = 12z$, for every i=1,2
As well as
$\sigma(vi) = 16*(iz)$, for every i=1,2,3
$\sigma(v_{i+1}) - \sigma(v_i) = 13z$, for every i=1,2
and
$\sigma(v_i) = 17*(iz)$, for every i=1,2,3
$\sigma(v_{i+1}) - \sigma(v_i) = 17z$, for every i=1,2
and
$\sigma(v_i) = 18*(iz)$, for every i=1,2,3
$\sigma(v_{i+1}) - \sigma(v_i) = 18z$, for every i=1,2
and
$\sigma(v_i) = 19*(iz)$, for every i=1,2,3
$\sigma(v_{i+1}) - \sigma(v_i) = 19z$, for every i=1,2
In all these above cases $\sigma(v_i)+\mu(v_iv_{i+1})+\sigma(v_{i+1})\leq 1$ is distinct for every i=1,2,...n-1 and which belongs to (0,1].
Therefore for every $3\leq n\leq 17$, the path graph P_n is a fuzzy antimagic graph with equal difference of vertices.
Theorem :2
Every fuzzy antimagic path graph P_n is graceful when $3\leq n\leq 7$ with some common difference of edges.
Proof:
Let $v_1,v_2,v_3...v_n$ be the vertices and $e_1,e_2,e_3...e_{n-1}$ be the edges of the path graph P_n.
Let $z\rightarrow(0,1]$ such that one can choose z=0.01 if $n\leq 7$ and z=0.001 if n7
Consider the following cases.
Case(i): $\mu(v_{i+1},v_{i+2}) - \mu(v_i,v_{i+1}) = 2z$
When the edge difference is 2z, the fuzzy antimagic path graph P_n will be graceful for every $3\leq n\leq 7$.
Here
$\mu(v_{i+1},v_{i+2}) - \mu(v_i,v_{i+1}) = 2z$, z=0.01 and i=1,2,...n-2 and
$\mu(v_i,v_{i+1}) = |\sigma(v_i)-\sigma(v_{i+1})| = 2iz$, i=1,2,...n-1
Moreover
$\sigma(v_i)+\mu(v_iv_{i+1})+\sigma(v_{i+1})$ is distinct for every i=1,2,...n-1.

*Example***:**



Fuzzy graceful and antimagic path graph – P_7

Case(ii): $\mu(v_{i+1,}v_{i+2})$- $\mu(v_{i,}v_{i+1})$ =3z

When the edge difference is3z, the fuzzy antimagic path graph P_n will be graceful for every $3 \le n \le 6$. Here

$\mu(v_{i+1,}v_{i+2})$- $\mu(v_{i,}v_{i+1})$ =3z, z=0.01 and i=1,2,...n-2 and

$\mu(v_{i,}v_{i+1}) = |\sigma(v_i)-\sigma(v_{i+1})|$ =3iz , i=1,2,...n-1

*Case (iii)***:** $\mu(v_{i+1,}v_{i+2})$- $\mu(v_{i,}v_{i+1})$ =4z

When the edge difference is 4z, the fuzzy antimagic path graph P_n will be graceful for every $3 \le n \le 5$. Here

$\mu(v_{i+1,}v_{i+2})$- $\mu(v_i,v_{i+1})$ =4z, z=0.01 and i=1,2,...n-2 and

$\mu(v_{i,}v_{i+1}) = |\sigma(v_i)-\sigma(v_{i+1})|$ =4iz , i=1,2,...n-1

*Case(iv)***:** $\mu(v_{i+1,}v_{i+2})$- $\mu(v_{i,}v_{i+1})$ =5z or 6z or 7z

When the edge difference is 5z or 6z or 7z, the fuzzy antimagic path graph P_n will be graceful for every $3 \le n \le 4$.

Case(v): $\mu(v_{i+1,}v_{i+2})$- $\mu(v_{i,}v_{i+1})$ = 8z or 9z or 10z or 11z or 12z

When the edge difference is 8z or 9z or 10z or 11z or 12z , the fuzzy antimagic path graph P_n will be graceful for n=3

In all these above cases,

$\sigma(v_i)+\mu(v_iv_{i+1})+\sigma(v_{i+1})$ is distinct for every i=1,2,...n-1.

So the fuzzy antimagic path graph P_n will be graceful for every $3 \le n \le 7$.

Theorem :3

The fuzzy antimagic butterfly graph B_n is graceful when n=7 or 9 with z=0.01.

Proof :

When n=7 the number of edges will be 8 and when n=9 the number of edges will be 12 in the butterfly graph B_n.

Consider V_1 as a common vertex for all edges in the following butterfly graph B_9. Here

$\mu(v_1,v_2) = \sigma(v_1)-\sigma(v_2)$ =0.01

$\mu(v_1,v_3) = \sigma(v_1)-\sigma(v_3)$ =0.03

$\mu(v_1,v_4) = \sigma(v_1)-\sigma(v_4)$ =0.06

$\mu(v_1,v_5) = \sigma(v_1)-\sigma(v_5)$ =0.10

$\mu(v_1,v_6) = \sigma(v_1)-\sigma(v_6)$ =0.15

$\mu(v_1,v_7) = \sigma(v_1)-\sigma(v_7)$ =0.21

$\mu(v_1,v_8) = \sigma(v_1)-\sigma(v_8)$ =0.28

$\mu(v_1,v_9) = \sigma(v_1)-\sigma(v_9)$ =0.36

Also

$\mu(v_4,v_5) = \sigma(v_4)-\sigma(v_5)$ =0.04

$\mu(v_5,v_6) = \sigma(v_5)-\sigma(v_6)$ =0.05

$\mu(v_7,v_8) = \sigma(v_7)-\sigma(v_8)$ =0.07

$\mu(v_8,v_9) = \sigma(v_8)-\sigma(v_9)$ =0.08

In all these relations, the butterfly graph B_9 satisfies

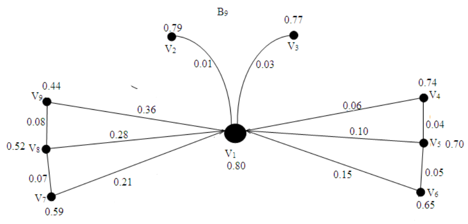
(i). $\mu(u,v) \le \sigma(u)$ □ $\sigma(v)$

(ii). the membership value of edges and vertices are distinct and $\mu(u,v)$ □ $\sigma(u)$ □ $\sigma(v)$ for all u,v ϵ V.

(iii). For any two distinct vertices u and v, the sum of the labels on edges incident to u is different from the sum of the labels on edges incident to v.

ie, $\mu(v_i,v_{i+1}) + \sigma(v_{i+1}) + \mu(v_{i+1},v_{i+2})$ is distinct for all i=4,5,...n-2.

Example:



Fuzzy graceful and antimagic butterfly graph – B_9

III. Conclusion:

In this paper the concept of fuzzy antimagic path graphs and butterfly graphs are introduced. We are planning to extend this for bistar graphs.

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