RESULT ON TOPOLOGIZED AND NON TOPOLOGIZED GRAPHS

S. Samboorna[#1], S. Vimala[*2]

[#1] *PG Student Department of Mathematics,*[*2] *Assistant Professor Department of Mathematics,*
[1,2,]*Mother Teresa Women's University Kodaikanal.*

[#1] sambus208@gmail.com
[*2]tvimss@gmail.com

***Abstract*— Topological graph theory deals with embedding the graphs in Surfaces, and the graph considered as a topological spaces. The concept topology extended to the topologized graph in[2]. This paper examines some results about the topologized and non topologized graphs and its subgraphs approach. The result were generalized and analyzed.**

***Keywords*— Complete, Path, Cycle, Cyclic, Topology.**

I. INTRODUCTION

The topologized graphs defined to R. Diesteland D. Kuhn. [5] “Topological Paths,Cycles in infinite graphs”, Europ, J.Combin., 2004. Antoine Vella [3]. In 2005 Antoine Vellatried to express combinatorial concepts in topological language. As a part the investigation classical topology, pre path, path [2], pre cycle, compact space, cycle space and bond space [3], locally connectedness and ferns were defined. For Topological graph theory, more important application can be found in printing electronic circuits where the aim is to print (embed) a circuit on a circuit board without two connections crossing each other and resulting in a short circuit. The great personality of mathematics Whitney expressed planarity in terms of the existence of dual graphs. A graph is planar if and only if it has a dual. In addition to this, planarity and the other related concepts are useful in many practical situations. For example, in the design of a printed – circuit board and the three utilities problem, planarity is used. There have been several alternative characterizations of planar graphs. In this paper we have verify the topologizedand non topologizedgraphs like as Sun graph, Sunlet graph, Fan graph. All graphs are finite, simple, undirected graphs with no loops and multiple edges and planar. Every topological space considered here are finite. And the topology is defined on the set X which is the union of vertices (V) and edges (E) of the graph G.

II PRELIMINARIES

2.1 Topological Space

A topology on a set X is a collection τ of subset of X with the following properties:

i. ϕ and X are inτ.

ii. The union of the element of any sub collection of τ is in τ (arbitrary union)

iii. The intersection of the element of any finite sub-collection of τ is inτ.

The set X for which a topology τ has been specified is called a topological space.

2.2Topologized graph

A topologized graph is a toplogical space X such that

- Every singleton is open or closed.
- For all $x \in X$, $|\partial(x)| \leq 2$.

2.3 Sun graph

A Sun graph as a graph no nodes consisting of a central complete graph with an outer ring of vertices, each of which is joined to both end points of the closet outer edge of the central core.

2.4 Sunlet graph

The n-sunlet graph is the graph on 2n vertices obtained by attaching, pendent edges to a cycle graph C_n.

2.51- regular graph

A one regular graph consist of disconnected edges.

2.6 Cycle graph

A simple graph with ‘n’ vertices (n>-3) and ‘n’ edges is called a cycle graph if all its edges from a cycle of length ‘n’. If the degree of each vertex in the graph is two, then it is called a Cycle Graph.

Notation: Cn

2.7 Cyclic graph

A cyclic graph is a graph containing atleast one graph cycle.

A graph that is not cyclic is said to be acyclic.

2.8 Path graph

A path graph is therefore a graph that can be drawn so that all of its vertices and edges lie on a single straight line.

2.9 Complete graph

A simple graph in which there exists an edge between every pair of vertices is called a Complete Graph.

2.10 Planar graph

A graph is planar if it can be drawn in a plane without graph edges crossing.

III MAIN RESULT

Theorem: 3.1

Show that sunlet graph G is not a topologized graph.

Proof:

Let G is a sunlet graph with the topology defined by $V \cup E$. Let (X, τ) be a topologized space. Using method of induction, For n=6, |X =12 the graph consists six vertices and six edges. The boundary of each edges is two and the boundary of three vertices in one and the boundary of three vertices in three. Since it doesnot satisfied topologized condition.Therefore, the sunlet graph is not a topologized graph.

Example: 3.1.1

Let G be a sunlet graph.

Vertex set V = $\{v_1,v_2,v_3,v_4,v_5,v_6\}$ and edge set E = {a,b,c,d,e,f}.

The edges are labeled as $f_G(a) = \{v_1, v_2\}$, $f_G(b) = \{v_3, v_4\}$, $f_G(c) = \{v_3, v_4\}$, $f_G(d) = \{v_3, v_5\}$,

$f_G(e) = \{v_5, v_6\}$, $f_G(f) = \{v_2, v_5\}$

Let X = $\{v_1, v_2, v_3, v_4, v_5, v_6, a, b, c, d, e, f\}$ be a topological space.

The topology defined by $\tau = \{X, \emptyset, \{V_1\}, \{V_2\}, \{a\}, \{V_1, V_2\}, \{V_1, a\}, \{V_2, a\}, \{V_1, V_2, a\}\{V_1, V_2, a, b\}\}$. and every singleton is open or closed.

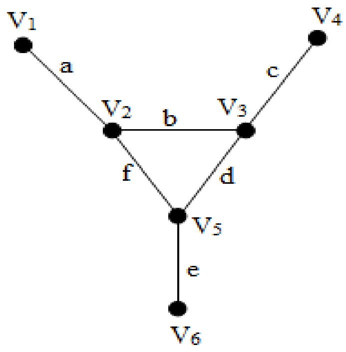

Fig.1

$\partial(v_1) = \{v_1\}$, so that $|\partial(v_1)| = 1$; $\partial(v_2) = \{v_1, v_3, v_5\}$, so that $|\partial(v_2)| = 3$;

$\partial(v_3) = \{v_2, v_4, v_5\}$, so that $|\partial(v_3)| = 3$; $\partial(v_4) = \{v_3\}$, so that $|\partial(v_4)| = 1$;

$\partial(v_5) = \{v_2, v_3, v_6\}$, so that $|\partial(v_5)| = 3$; $\partial(v_6) = \{v_5\}$, so that $|\partial(v_6)| = 1$;

$\partial(a) = \{v_1, v_2\}$, so that $|\partial(a)| = 2$; $\partial(b) = \{v_2, v_3\}$, so that $|\partial(b)| = 2$;

$\partial(c) = \{v_3, v_4\}$, so that $|\partial(c)| = 2$; $\partial(d) = \{v_3, v_5\}$, so that $|\partial(d)| = 2$;

$\partial(f) = \{v_5, v_6\}$, so that $|\partial(f)| = 2$;

It is not satisfied the topologized condition. Therefore, the sunlet graph is not a topologized graph.

Theorem: 3.2

Show that sunlet graph is not topologized but every subgraph (1-regular graph, cycle graph C_3) of a sunlet graph is topologized.

Proof:

Let a sunlet graph is not topologized.

To prove,

Every subgraph of a graph is topologized. Let us take a sunlet graph figure.1

The sunlet graph is not satisfied the topologized condition. So, is not a topologizedgraph.

Let us take a subgraph (1- regular graph) of the above graph,

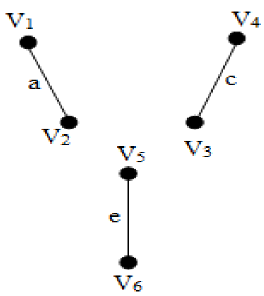

Fig. 2

The boundary value of the graph,

$\partial(v_1) = \{v_2\}$, so that $|\partial(v_1)| = 1$; $\partial(v_2) = \{v_1\}$, so that $|\partial(v_2)| = 1$;

$\partial(v_3) = \{v_4\}$, so that $|\partial(v_3)| = 1$; $\partial(v_4) = \{v_3\}$, so that $|\partial(v_4)| = 1$;

$\partial(v_5) = \{v_6\}$, so that $|\partial(v_5)| = 1$; $\partial(v_6) = \{v_5\}$, so that $|\partial(v_6)| = 1$;

$\partial(a) = \{v_1, v_2\}$, so that $|\partial(a)| = 2$; $\partial(c) = \{v_3, v_4\}$, so that $|\partial(c)| = 2$;

$\partial(e) = \{v_5, v_6\}$, so that $|\partial(e)| = 2$;

It is satisfied the topologized condition. Therfore, the subgraph (1-regular graph) is topologized graph.

Again let us take a subgraph (cycle graph C_3).

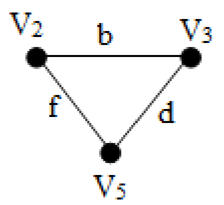

Fig.3

The boundary value of the graph,

$\partial(v_2) = \{v_3, v_5\}$, so that $|\partial(v_2)| = 2$; $\partial(v_3) = \{v_2, v_5\}$, so that $|\partial(v_3)| = 2$;

$\partial(v_5) = \{v_2, v_3\}$, so that $|\partial(v_5)| = 1$;

$\partial(b) = \{v_2, v_3\}$, so that $|\partial(b)| = 2$; $\partial(d) = \{v_3, v_5\}$, so that $|\partial(d)| = 2$;

$\partial(f) = \{v_2, v_5\}$, so that $|\partial(f)| = 2$;

It is satisfied the topologized condition. So, it is a topologized graph.

Theorem: 3.3

Show that the sunlet graph is not topologized and the subgraph(planer graph) is topologized.

Proof:

Let us take a sunlet graph.

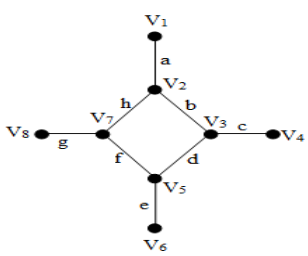

Fig. 4

$\partial(v_1) = \{v_2\}$, so that $|\partial(v_1)| = 1$; $\partial(v_2) = \{v_1, v_3, v_7\}$, so that $|\partial(v_2)| = 3$;

$\partial(v_3) = \{v_2, v_4, v_5\}$, so that $|\partial(v_3)| = 3$; $\partial(v_4) = \{v_3\}$, so that $|\partial(v_4)| = 1$;

$\partial(v_5) = \{v_3, v_6, v_7\}$, so that $|\partial(v_5)| = 3$; $\partial(v_6) = \{v_5\}$, so that $|\partial(v_6)| = 1$;

$\partial(v_7) = \{v_2, v_5, v_8\}$, so that $|\partial(v_7)| = 3$; $\partial(v_8) = \{v_7\}$, so that $|\partial(v_8)| = 1$;

$\partial(a) = \{v_1, v_2\}$, so that $|\partial(a)| = 2$; $\partial(b) = \{v_2, v_3\}$, so that $|\partial(b)| = 2$;

$\partial(c) = \{v_3, v_4\}$, so that $|\partial(c)| = 2$; $\partial(d) = \{v_3, v_5\}$, so that $|\partial(d)| = 2$;

$\partial(e) = \{v_5, v_6\}$, so that $|\partial(e)| = 2$; $\partial(f) = \{v_5, v_7\}$, so that $|\partial(f)| = 2$;

$\partial(g) = \{v_7, v_8\}$, so that $|\partial(g)| = 2$; $\partial(h) = \{v_2, v_7\}$, so that $|\partial(h)| = 2$;

It is not satisfied the topologized condition. So, the sunlet graph is not topologized.

Let us take a subgraph (planar graph),

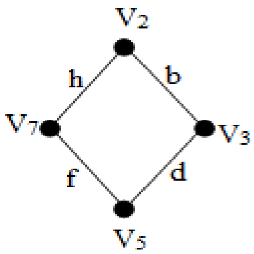

Fig. 5

$\partial(v_2) = \{v_3, v_7\}$, so that $|\partial(v_2)| = 2$; $\partial(v_3) = \{v_2, v_5\}$, so that $|\partial(v_3)| = 2$;

$\partial(v_5) = \{v_3, v_7\}$, so that $|\partial(v_5)| = 2$; $\partial(v_7) = \{v_2, v_5\}$, so that $|\partial(v_7)| = 2$;

$\partial(b) = \{v_2, v_3\}$, so that $|\partial(b)| = 2$; $\partial(d) = \{v_3, v_5\}$, so that $|\partial(d)| = 2$;

$\partial(f) = \{v_5, v_7\}$, so that $|\partial(f)| = 2$; $\partial(d) = \{v_2, v_7\}$, so that $|\partial(h)| = 2$;

It is satisfied the topologized condition. So, that the planar graph is topologized.

Hence, the sunlet graph is not topologized but the subgraph of a planar graph is topologized.

Theorem: 3.4

Show that sungraphG isa not topologized graph.

Proof:

Let G is a sungraph with the topology defined $V \cup E$. Let (X,τ) be a topological space. Using the induction method, For n = 6, $|X| = 15$ the graph consists six vertices and nine edges. The boundary of each edge is two and the boundary of three vertices is two and the boundary of three vertices is four. It is not satisfied the topologized condition. So, the sun graph is not topologized graph.

Example: 3.1.2

Let G be a sun graph. Vertex V = {v_1, v_2, v_3, v_4, v_5, v_6} and edge set E = {a, b, c, d, e, f, g, h, i}.

The edge are labeled as $f_G(a)$ = {v_1, v_2}, $f_G(b)$ = {v_2, v_3}, $f_G(c)$ = {v_3, v_4}, $f_G(d)$ = {v_4, v_5}, $f_G(e)$ = {v_5, v_6}, $f_G(f)$ = {v_1, v_6}, $f_G(g)$ = {v_2, v_6}, $f_G(h)$ = {v_4, v_6}, $f_G(i)$ = {v_2, v_4}. Let X = {v_1, v_2, v_3, v_4, v_5, v_6, a, b, c, d, e, f, g, h, i} be a topological space. The topology defined by τ = {X, $\emptyset$, {V_1}, {a}, {b}, {V_1, a}, {V_1, b}, {a, b}, {V_1, a, b}, {V_1, V_2, V_3, a, b}}. And every singleton is open or closed.

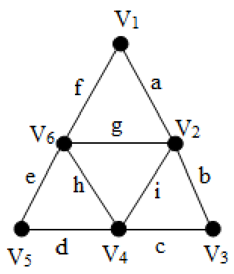

Fig. 6

$\partial(v_1)$ = {v_2, v_6}, so that | $\partial(v_1)$ | = 2;
$\partial(v_2)$ = {v_1, v_3, v_4, v_6}, so that | $\partial(v_2)$ | = 4;
$\partial(v_3)$ = {v_2, v_4}, so that | $\partial(v_3)$ | = 2;
$\partial(v_4)$ = {v_2, v_3, v_5, v_6}, so that | $\partial(v_4)$ | = 4;
$\partial(v_5)$ = {v_4, v_6,}, so that | $\partial(v_5)$ | = 2;
$\partial(v_6)$ = {v_1, v_2, v_4, v_5}, so that | $\partial(v_6)$ | = 4;

$\partial(a)$ = {v_1, v_2}, so that | $\partial(a)$ | = 2;
$\partial(b)$ = {v_2, v_3}, so that | $\partial(b)$ | = 2;
$\partial(c)$ = {v_3, v_4}, so that | $\partial(c)$ | = 2;
$\partial(d)$ = {v_4, v_5}, so that | $\partial(d)$ | = 2;
$\partial(e)$ = {v_5, v_6}, so that | $\partial(e)$ | = 2;
$\partial(f)$ = {v_1, v_6}, so that | $\partial(f)$ | = 2;
$\partial(g)$ = {v_2, v_6}, so that | $\partial(g)$ | = 2;
$\partial(h)$ = {v_4, v_6}, so that | $\partial(h)$ | = 2;
$\partial(i)$ = {v_2, v_4}, so that | $\partial(i)$ | = 2;

It is not satisfied the topologized condition. Therefore, the sunlet graph is not a toplogized graph.

Theorem:3.5

Show that in a graph is not topologized and the subgraph (path graph, planar graph, cyclic graph) of a sun graph is either topologized or not topologized.

Proof:

Let G be a sun graph, figure: 1 shows that it is not a topologized graph.

Let us take a subgraph (path graph) of a sun graph,

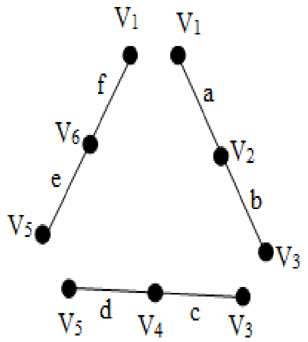

Fig. 7

It is satisfied the topologized condition. Therefore, the path graph P_3 is a topologizedgraph.Again take a subgraph (planar graph),

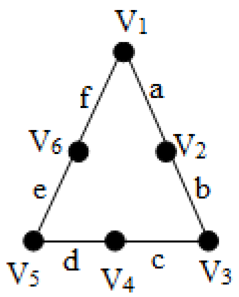

Fig. 8

It is satisfied the topologizedcondition.So, the planar graph is a topologized graph.

Again let us a take a subgraph (cyclic graph),

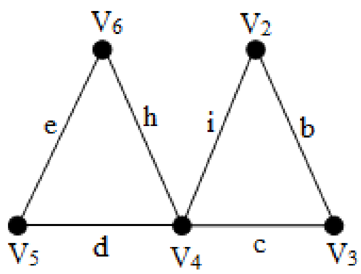

Fig. 9

The subgraph contains more than one cycle. So, it is called a cyclic graph. Then the cyclic graph is not satisfied the topologized condition for $|\partial(v_4)| = \{v_2, v_3, v_5, v_6\} = 4$. So, it is not a topologized graph.

Hence, the sungarph is not topologized and the subgraphs are either topologized or nor topologized.

Theorem: 3.6

Show that the sungraph is not topologized and the subgraph (complete graph) is the non-topologized.

Proof: Let us take a sun graph,

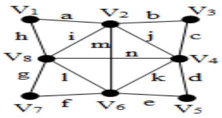

Fig. 10

The above graph is not satisfied the topologized condition. So, the sungraph is not topologized.

Let us take a supgraph (complete graph C_3),

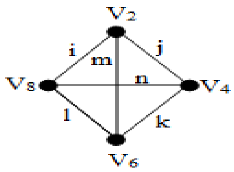

Fig. 11

Then it is not satisfied the topologized condition. So, the complete graph K_4 is not topologized graph.

Hence, the sun graph is not topologized and the subgraph (complete graph K_4) is also not topologized graph.

IV CONCLUSION

In this paper some mew results were found about the topologized graph and non topologized graphs and its subgraphs through the Sun graph, Sunlet graph.

REFERENCES

[1] Armen S. Asratian. LuleåTekniskaUniversitet, Sweden, Tristan M.J. Denleyuniversity of

Mississippi, Roland Haggkvist, UmeåUniversitet, Sweden, ‖Bipartite Graphs and their Applications; 2008.

[2] Arumugam S, Ramachandran S. Invitation to graph theory. Published by Scitech Publications (India) Pvt Lt; 2006.

[3] Antoine Vella. A fundamendally topological perspective on graph theory. Ph.D., Thesis, Waterloo, Ontario, Canada; 2005.

[4] Berge C. Hypergraphs. Combinatories of finite sets. Number 45 in North –Holland Mathematical Library. North Holland Publishing Co., Amsterdam. (Translated from the French); 1989.

[5] Diestel R, Kuhn D. Topological paths, cycles in infinite graphs. Europ. J. Combin.; 2004.

[6] Diestel R. The cycle space of an infinite graph. To Appear in Combin. Probab. Comput.

[7] Eppstein David. Testing bipartiteness of geometric intersection graphs. ACM Transactions on Algorithms; 2009.

[8] Goerge F. Simmons. Introduction to topology and modern analysis. Published by Robert E. Kringer Publishing Company Inc. Malabar, Floria © 1963 by McGraw – Hill, Inc. ISBN – 0 – 89874 – 551- 9

[9] ReinhardDiestel. Graph theory. Graduate texts in mathematics. Springer- Verlag New Yark. 2000; 173.

[10] Skiena S. Coloring bipartite graphs. §5.5.2. In: Skiena S (ed) Implementing discrete mathematics: Combinatorics and Graph theory with Mathematica; 1990.