INTRODUCTION ON BIPOLAR SOFT MULTISET TOPOLOGICAL SPACES

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***Abstract*—Due to the absence of adequacy of parameterization concepts we slightly move from classical mathematics to modern mathematics for solving uncertainty problems occurs in engineering, environmental science, medical science, physics, chemistry and so many fields. At present Fuzzy sets, rough sets, soft sets and combination of these three sets are acting as the emerging mathematical tools. Many researchers in mathematics initiated so many hybrid versions of the above sets. Bipolar soft set was constructed by bipartition soft set, was initiated by Faruk et al.[9]. Softmultiset was introduced by Babithaet.al[6]. In this paper let us see the concept of bipolar soft multiset and its operations.**

***Keywords*— bipolar soft multiset, multiset, soft multiset, soft set**

I. INTRODUCTION

Numerous entanglements are occurring to take a decision in so many situations in the field of engineering, medicine science, environmental science and so some other fields. Molodtsov [4] defined the function with the non-numerical data that helps to overcome an uncertainty problems and vagueness problems. After his work many researchers in Mathematics and computer Science fabricated their own ideas on soft set theory and enrich its progress. In decision making we able to use soft set easily. Soft multi set was registered by Babitha et.al [6]and bipolar soft set was introduced by Faruket.al [9] which differs from bipolar fuzzy soft theory. Bipolar soft set is constructed by the bipartition on attribute set of the soft set. Gilbert et.al [] introduced the concept of bipolar soft multiset theory and investigated its properties. In this paper we introduce bipolar soft multiset topological space and define some basic ideas for it.

The following chapters contain the following sequential numbers. Section 2 helps to recollect the basic definitions, section3 introduces bipolar soft multiset topological space, section 4 concludes this paper.

II. PRELIMINARIES

Throughout this paper, we will denote initial universe set of parameters and power set of U, E and P(U)respectively.
In this section we recollect the definitions and rotations as mentioned by Molodstov[4] and Majiet. al [11]. We also recall definition of soft multiset[6].

Definition 2.1. Let $\mathbb{U}$ be an initial universe set and E be a set of parameters. LetP(U) denotes the power set of U and A $\subseteq$E. A pair (F, A) is called a soft set over $\mathbb{U}$, where F is a mapping given by F : A→P ($\mathbb{U}$)

Definition 2.2. Let A = $\{e_1, e_2 \dots \dots e_n\}$ be a set of parameters. The NOT set of A denoted by ⌉A is defined by ⌉A = $\{⌉e_1, ⌉e_2, \dots ⌉e_n\}$ where $⌉e_i$= not e_i, ∀ I = 1, 2,……n.

Definition 2.3 Let $\mathbb{U}$ be universal mset and E be set of parameters. Then an ordered pair (F, E) is called a soft multiset where F is a mapping given by F: A→PW ($\mathbb{U}$),PW ($\mathbb{U}$) is defined as the set of all whole submset of M.

Definition 2.4. Let M be a multiset. Then the relative compliment of a whole submset M_1 of M is given by M_1^r= ${}^{m_i}/_{x_i}$ where $C_M^{(xi)}$ = 0 Forevery x_i in M_1 and m_i is the count of x_i in M.

Definition 2.5. Let Ebe a parameters set and E_1, E_2 be two nonempty subsets of E such that $E_1 \cup E_2$ = E and $E_1 \cap E_2$ = Φ If F : E_1 →PW ($\mathbb{U}$) and G: E_2 →PW ($\mathbb{U}$) are two mappings such that $F(e) \cap G(f(e))$= Φ then triple (F,G,E) is called bipolar soft multiset where F: $E_1 \to E_2$ is a bijective function. Set of all bipolar soft multisets over $\mathbb{U}$ is denoted by BMS. We can represent a bipolar soft multiset (F, G, E) as follows.

$$(F, G,E) = \left\{ \begin{matrix} < \left(e, F(e), \left(f(e), G1f1(e)\right)\right) > \\ e \in E_1 \ and \ F(e) \cap G(f(e)) = \Phi \end{matrix} \right\}$$

III. BIPOLAR SOFT MULTISET TOPOLOGICAL SPACES

In this section, we will introduce bipolar soft multiset topological spaces.

Definition 3.1

Let $U=\{u_1,u_2,\ldots,u_k\}$ then $[U]^w=\{\{k_1/u_1,k_2/u_2,\ldots\ldots,k_n/u_n\}: for\ i=1,2,\ldots\ldots,k;\ K_i\in\{0,1,2,\ldots\ldots,w\}\}$. Let E be a parameters set E_1, E_2 be two nonempty subsets of E such that $E_1\cup E_2=E$ and $E_1\cap E_2=\phi$. Let (F,G,E) be a bipolar soft multiset . Let $\tilde{\tau}^{C_m}\subseteq P^*((F,G,E))$. Then $\tilde{\tau}^{C_m}$ is called a BSM-topology of (F, G, E) over U if $\tilde{\tau}^{C_m}$ satisfies the following:

1. $(\phi,U,E),(F,G,E)\in\tilde{\tau}^{C_m}$
2. $(F,G,E),(F,K,E)\in\tilde{\tau}^{C_m}$ then $(F,G,E)\cap(H,K,E)\in\tilde{\tau}^{C_m}$
3. If (F_i,G_i,E) for every $i\in I$ then $\cup\{G_i,A):i\in I\}\in\tilde{\tau}^{C_m}$

The triplet $(F,\tilde{\tau}^{C_m},E)$ is called a Bipolar Soft multiset topological space or BSM-topological space(shortly BSM$(F,\tilde{\tau}^{C_m},E)$).

Remark 3.2:

The members of $\tilde{\tau}^{C_m}$ are called bipolar soft open multisets in U. Also a bipolar soft multiset is called bipolar soft closed multiset if the complement $(F,G,E)^{\tilde{C}_m}$ belongs to $\tilde{\tau}^{C_m}$. The family of bipolar soft closed multisets is denoted by C$\tilde{\tau}^{C_m}$.

Theorem 3.3:

Let $(F,\tilde{\tau}^{C_m},E)$ be a bipolar soft topological space and the following hold:

1. $(\phi,U,E)',(F,G,E)\in C\tilde{\tau}^{C_m}$
2. $(F_1,G_1,E),(F_2,G_2,E)\in\tilde{\tau}^{C_m}$ then $(F_1,G_1,E)\cup(F_2,G_2,E)\in C\tilde{\tau}^{C_m}$
3. If (F_i,G_i,E) for every $i\in I$ then $\cap\{(F_i,G_i,A):i\in I\}\in C\tilde{\tau}^{C_m}$

Definition 3.4:

Let $(F,\tilde{\tau}^{C_m},E)$ be a bipolar soft multiset topological space and (F,G,A) $\subseteq$ (F,G,E). Then the bipolar soft minterior of a bipolar soft mset (F,G, A) is denoted by $(F,G,A)^0$ and is defined as the Bipolar soft munion of all bipolar soft open subsets of (F,G,A). Thus $(F,G,A)^0$ is the largest bipolar soft mopen set contained in (F,G,A).

Theorem 3.5

Let $(F,\tilde{\tau}^{C_m},E)$ be a bipolar soft M-topological space and (F,G, A),(F,G,B) $\subseteq$ (F,G,E) then

(1) $(\phi,U,E)^o=(\phi,U,E)$

(2) $(F,G,A)^o\subset(F,G,A)$

(3) $((F,G,A)^o)^o\subset(F,G,A)^o$

(4) (F,G,B) is a bipolar soft mopen set if and only if $(F,G,A)^o=(F,G,A)$

(5) $(F,G,A)\subseteq(F,G,B)\Rightarrow(F,G,A)^o\subseteq(F,G,B)^o$

(6) $(F,G,A)^o\cap(F,G,B)^o=((F,G,A)\cap(F,G,B))^o$

(7) $(F,G,A)^o\cup(F,G,B)^o\subseteq((F,G,A)\cup(F,G,B))^o$

Definition 3.6:

Let $(F,\tilde{\tau}^{C_m},E)$ be a bipolar soft topological space and (F,G,A) $\subseteq$ (F,G,E). Then the bipolar soft closure of (F,G, A) is denoted by $\overline{(F,G,A)}^m$ is defined as the soft mintersection of all bipolar soft mclosed supermsets of (F,G, A). $\overline{(F,G,A)}^m$ is the smallest bipolar soft mclosed set containing (F,G,A).

Theorem 3.7:

Let $(F,\tilde{\tau}^{C_m},E)$ be a bipolar soft M- topological space and (F,G,A) $\not\subset$ (F,G,E). (F,G,A) is a bipolar soft mclosed set if and only if $\overline{(F,G,A)}=(F,G,A)$

Theorem 3.8:

Let $(F,\tilde{\tau}^{C_m},E)$ be a bipolar soft M-topological space and (F,G,A),(F,G,B) $\subseteq$ (F,G,E). Then

$$(1)\,\overline{\overline{(F,G,A)}}=\overline{(F,G,A)}$$

$$(2)\,(F,G,B)\subset(F,G,A)\Longrightarrow\overline{(F,G,B)}\subset\overline{(F,G,A)}$$

$$(3)\,\overline{(F,G,A)\cup(F,G,B)}\supset\overline{(F,G,A)}\cup\overline{(F,G,B)}$$

$$(4)\overline{(F,G,A)\cap(F,G,B)}\supseteq\overline{(F,G,A)}\cap\overline{(F,G,B)}$$

Definition 3.9:

Let $(F,\tilde{\tau}^{C_m},E)$ be a bipolar soft M-topological space and *(F,G,A)* $\subseteq$ *(F,G,E).* Then, the bipolar soft mboundary of bipolar soft mset *(F,G, A)* is denoted by *(F,G,A)*^{mb} and is defined as $(F,G,A)^{bm}=\overline{(F,G,A)}\cap\overline{(F,G,A)^{C}}$

Theorem 3.10

Let $(F,\tilde{\tau}^{C_m},E)$ be a bipolar soft topological space and (F,G,A),(F,G,B) $\subseteq$ (F,G,E). Then

$$(1)(F,G,B)^{bm}\subseteq\overline{(F,G,B)}$$

$$(2)(F,G,B)^{bm}=((F,G,B)^{C})^{bm}$$

$$(3)(F,G,B)^{bm}=\overline{(F,G,B)}\setminus(F,G,B)^{O}$$

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